Oil Consumption, Economic Growth, and Oil Futures:
The Impact of Long-Run Oil Supply Uncertainty on Asset Prices

Internet Appendix: Not For Publication

Robert Ready

First Draft: July 3, 2014
This Draft: February 26, 2016

Appendix A: Data Sources

Data on total U.S. oil consumption and worldwide oil production comes from the Energy Information Association (EIA). Data on household consumption and GDP come from the BEA’s NIPA surveys. Data on TFP, hours worked, and capital supply are from the San Francisco Federal Reserve.\(^1\) Data on oil futures are for the NYMEX West Texas Intermediate (WTI) contract, and come from the Commodity Research Bureau.\(^2\) All data are for the U.S. economy. Data on miles per gallon of the U.S. passenger car fleet are from the National Transportation Safety Board.

Data on oil price forecasts come from the European Central Bank’s Survey of Professional Forecasters. Data on news articles comes from Factiva, and internet search data comes from Google Trends. Industry returns are taken from Ken French’s website.

The macroeconomic data and oil spot price are typically available for longer time series. To be consistent with other macroeconomic studies of oil prices, I report data for 1970-2012. The structural change in open interest generally attributed to financialization is usually identified as occurring near the end of 2004 (Hamilton and Wu (2013)).

Appendix B: Model Equilibrium Conditions

The model is defined by the specifications for utility and the production technology, as well as the constraints for capital accumulation, the labor supply, and storage. Here I present the extended model which includes a storage technology. The results for this model are shown in Appendix D.

The representative agent’s utility is given by

\(^1\)http://www.frbsf.org/economics/economists/jfernald/quarterly_tfp.xls

\(^2\)Since 2011 there has been a divergence between the WTI and other global oil price indices. In unreported analyses, the tests shown in this section were repeated using Brent Crude futures and yielded qualitatively similar results.
\[ U_t = \left[ (1 - \beta) \left( \bar{C}_t^{1-\phi} N_t^{\phi} \right)^{1-\frac{1}{\phi}} + \beta \left( E_t[U_{t+1}^{1-\gamma}] \right)^{1-\frac{1}{\gamma}} \right]^{\frac{1}{1-\phi}} \]  \tag{1}

Where \( N_t \) is leisure adjusted for increases in living standards (\( N_t = A_t l_{t-1} n_t \) where \( n_t \) is hours devoted to leisure), and \( \bar{C}_t \) is the intratemporal utility over the basic consumption good \( (C_t) \) and the consumption of the oil good \( (G_t) \) and has the form

\[ \bar{C}_t = \left[ (1 - a_G) C_t^{1-\frac{1}{\gamma}} + a_G G_t^{1-\frac{1}{\gamma}} \right]^{\frac{\gamma}{\gamma - 1}} \]

Production of the basic good used for consumption and investment is given by

\[ Y_t = \left[ (1 - a_O) \left( K_t^\alpha (A_t L_t)^{\alpha - 1} - \frac{1}{\gamma} + a_O O_t^{1-\frac{1}{\gamma}} \right]^{\frac{\gamma}{\gamma - 1}} \]

Where \( A_t \) is an exogenous shock, \( L_t \) is labor, \( O_t \) is oil used as an input to production, and \( K_t \) is the capital stock.

The constraints for investment \( (I_t) \), the labor supply, and capital accumulation are given by

\[ Y_t = C_t + I_t \] \tag{3}
\[ 1 = L_t + n_t \] \tag{4}
\[ K_{t+1} = (1 - \delta_k) K_t + \Phi(\frac{I_t}{K_t}) K_t \] \tag{5}

Where \( \Phi \) is an adjustment cost function parameterized as in Jermann (1998).

The constraint on oil use and storage is given by

\[ W_t + S_{t-1} = O_t + G_t + S_t + \Phi_S(S_t, H_t) \] \tag{6}

Where \( W_t \) is the exogenous supply of oil, and \( \Phi_S(S_t, H_t) \) is the cost of storing \( S_t \) units of oil, which depends on an exogenous supply of “Storage Capital” \( (H_t) \).

The social planner maximizes \( U_t \) subject to the constraints on production, capital, and oil storage. The first order conditions of the Lagrangian yield the following equilibrium conditions

The first order conditions for capital and investment yield the standard q-theoretic relation of the representative agent given by

\[ \frac{M_{t+1}}{M_t} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\frac{1}{\phi}} \left( \frac{\bar{C}_{t+1}/\bar{C}_t}{C_t/C_t} \right)^{-\frac{1}{\phi}} \left( \frac{N_{t+1}/\bar{C}_{t+1}}{N_t/\bar{C}_t} \right)^{(1-\frac{1}{\phi})(\phi)} \left( \frac{U_{t+1}}{E_t[U_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}} \right)^{\frac{1}{\gamma-1}} \] \tag{7}

The risk free rate is

\[ R_f^t = E[M_{t+1}]^{-1} \]

The first order conditions for capital and investment yield the standard q-theoretic relation
between current and future capital given by

\[ Q_t = E_t \left[ \frac{M_{t+1}}{M_t} Q_{t+1} \left( \frac{\partial Y_{t+1}}{\partial K_{t+1}} + (1 - \delta_k) + \Phi'(\frac{I_{t+1}}{K_{t+1}}) - \Phi'(\frac{I_{t+1}}{K_{t+1}}) \frac{I_{t+1}}{K_{t+1}} \right) \right] \] (8)

Where

\[ Q_t = \frac{1}{\Phi'(\frac{K_t}{K})} \] (9)

The first order conditions for leisure and labor yield the following relation

\[ \frac{\partial Y_t}{\partial L_t} = A_{t-1} \phi \frac{C_t}{C_t} \left( \frac{\tilde{C}_t}{\tilde{C}_t} \right)^{\frac{1}{a}} \] (10)

The first order conditions with respect to \( C_t \) and \( G_t \) imply that the spot price of oil is given by

\[ P_t = \frac{a_G}{(1 - a_G)} \left( \frac{C_t}{G_t} \right)^{\frac{1}{a_G}} \] (11)

The first order condition with respect to \( O_t \) gives

\[ P_t = \frac{\partial Y_t}{\partial O_t} \] (12)

The first order condition for the level of storage gives a relation between expected future prices and current prices

\[ \frac{E_t[M_{t+1} P_{t+1}]}{P_t} = R_{f,t} \left( 1 + \frac{d\Phi_S(S_t, H_t)}{dS_t} \right) \] (13)

To solve the model the equations (1) - (13) are normalized by the level of \( A_t \) to allow for a stationary representation of the model. The system is numerically solved for a deterministic steady state, and the dynamic equations as well as the dynamics for the exogenous state variables, are then used as equilibrium conditions in Dynare++ to solve the model. (See Croce (2014)).

Appendix C: Alternate Calibrations

In order to highlight the mechanisms necessary to generate the observed behavior in the futures curve, the benchmark model is recalibrated using two alternate scenarios. In the first, the impact of the oil supply on TFP growth is removed (\( \zeta = 0 \)) and in the second, \( \tilde{w} \) is increased so that oil expenditure relative to total output is roughly 50% of what it is in the Benchmark calibration. In both cases the model is again simulated in an unresponsive and responsive oil supply regime.

Table 1 shows the model moments under the two alternate scenarios as well as the Benchmark Model for comparison, and Figure 1 shows the term structures of oil futures.
As the table and figure show, the calibration without an exogenous impact of the oil supply on future TFP growth exhibits none of the increase in the term structure of average future prices that are observed in the data. In contrast, the scenario with an exogenous growth impact but more plentiful oil generates futures term structures similar to the benchmark model. This suggests that, when considering the behavior of oil futures, the persistence of price shocks may be more important than their aggregate level.

Appendix D - Model Extensions: Stochastic Volatility and Storage

This section extends the model on two dimensions. The first adds stochastic oil supply volatility and examines the impact on the variance risk premia associated with oil futures. The second adds a storage technology to the model.

D.1 Stochastic Oil Supply Volatility

Following Carr and Wu (2009) and Trolle and Schwartz (2010), a growing literature studies the pricing of variance or volatility risk in oil and energy markets. While stochastic oil volatility is not necessary to generate interesting futures curve dynamics, it is necessary to generate a variance risk premium. Here I include it in the model and show that in the presence of stochastic volatility, an unresponsive oil supply leads to higher variance risk premium, again consistent with the data.

D.1.1 Changes in Oil Variance Risk Premia

I follow the methodology of Trolle and Schwartz (2010) and construct prices for synthetic variance swaps for oil futures using options of varying moneyness, after accounting for the early exercise premium in American options. I then calculate the E(VRP), the expected variance risk premia, by using the difference between the price of the synthetic swap and the expected variance calculated using an ARMA(1,1) of realized variance. This expected return is calculated each month for a synthetic volatility swap on the the nearest maturity future. The expected return is calculated on the day the future is one month from expiration. I perform this procedure for copper and oil futures over the two time periods 1997 - 2004, and 2005 - 2012. Table 2 reports the results.

This increase in the variance risk premium for oil suggests that oil volatility was more expensive to insure in the post 2005 period. This finding is also related to the finding of Christoffersen and Pan (2014), who shows that from 2005 to 2012, the implied volatility of oil prices is a priced factor in equity markets, but that this effect is not present in earlier data.
D.1.2 Stochastic Volatility and Variance Risk in the Model

To add stochastic volatility I specify the following dynamics for \( w_t \) and \( v_t \).

\[
\begin{align*}
\Delta w_{t+1} &= \mu + (\rho_w - 1)(w_t - \bar{w}) + \kappa x_t + \exp(v_{w,t})\sigma_w \epsilon_{t+1}^w \quad (14) \\
v_{t+1}^w &= (\rho_v^w)v_t^w + \sigma_v^w \epsilon_{t+1}^v \quad (15)
\end{align*}
\]

I calibrate \( \rho_w \) and \( \sigma_v^w \) from an AR(1) of realized volatility. In the model shocks to oil volatility reinforce the shocks to levels in generating the observed futures effects, though they are not necessary. More interestingly, shocks to oil production volatility have a much larger impact on aggregate wealth when oil production is unresponsive to prices. This is illustrated in Figure 2, which shows the response of several model variables, including the stochastic discount factor, to an increase in oil volatility. This result follows from the intuition of Bansal and Yaron (2004) and Eraker (2008), who show that shocks to volatility are only important when shocks to the level have a significant price of risk. As a result, the increase in hedging premium generated by the increase in the persistence of oil prices also leads to an increased impact of shocks to oil price volatility, providing a potential explanation for the increase in the variance risk premium for oil, as well as for the findings of Christoffersen and Pan (2014). Christoffersen and Pan (2014) note that this finding is unique to oil, again suggesting that the impacts in oil market are a result of oil’s role as a fundamental input into the economy, rather than effects caused by increased trading in commodities.

[Figure 2 about here.]

D.2 Storage

Though oil storage does not drive any of the qualitative asset pricing implications of the model, it is nevertheless an important feature of the market that must be addressed by any model attempting to relate the dynamics of oil consumption to prices. In this section I report the basic relation of storage and the futures curve in the data, as well as the effects of adding storage to the model.

D.2.1 Oil Futures Prices and Storage Data

The basic theory of storage, introduced by Hotelling (1931) and developed by Deaton and Laroque (1992) and others, yields a simple relation between the marginal cost of storage and the slope of the futures curve.\(^3\) If storage costs are increasing in the level of storage, then this implies a positive relation between the level of oil inventories and the slope of the futures curve. This relation generally holds for oil prices in the data, as shown by Figure 3, which plots quarterly averages of oil stocks against the slope of the futures curve. As is clear from the graph, this relation is quite strong, with high levels of storage coinciding with an upward slope in the term structure of oil futures.

\(^3\)See Gorton, Hayashi, and Rouwenhorst (2013) for a detailed discussion of this relation.
[Figure 3 about here.]
D.2.2 Oil Storage in the Model

Storage is modeled in a reduced form way to capture the relation between storage and futures prices in the data. With oil inventories $S_t$, the oil resource constraint becomes

$$W_t + S_{t-1} = O_t + G_t + S_t + \Phi_S(S_t, H_t)$$  \hspace{1cm} (16)

$\Phi_S(S_t, H_t)$ represents the cost of storing a level $S_t$. $H_t$ represents “Storage Capital”. The log of storage capital $h_t$ evolves according to

$$\Delta h_{t+1} = \mu_z + (\rho_h - 1)(h_t - a_t - \bar{h})$$  \hspace{1cm} (17)

This variable is a separate very slow moving state variable to maintain stationarity, and is therefore not exposed to shocks. A convenient interpretation of this variable is that it represents the natural level of storage in the oil industry. While there are clear costs and limits to increasing storage, there are also potential costs to have too little storage on hand to meet sudden demands on inventory. Storage costs are therefore modeled as

$$\Phi_S(S_t, H_t) = \phi_0(S_t - H_t) + \phi_1 \frac{(S_t - H_t)^2}{H_t}$$  \hspace{1cm} (18)

The first term allows a constant marginal benefit or cost to additional storage. In the calibration $\phi_0$ is set so that the marginal benefit of storage is equal to the risk-free rate in the deterministic steady state when $S_t = H_t$, so that expected price growth is zero on the balanced growth path. The second term represents a quadratic cost to having storage away from its natural level $H_t$. $\phi_1 = .1$ is the quadratic cost parameters in the calibrations, and is set so that the response of the level of storage to changes in the futures curve is similar to the data. The steady state of storage capital is chosen so that the level of inventories is roughly equal to 15% of total annual oil consumption as in the data.

Given this specification, the level of storage is closely linked to the slope of the futures curve. This intuitive relation is generated in the model via the planner’s first order condition with respect to the storage choice variable $S_t$, which equates the slope of the futures curve to the marginal cost of storing an additional barrel of oil.

$$\frac{F_t}{P_t} = R_{f,t} \left( 1 + \frac{d\Phi_S(S_t, H_t)}{dS_t} \right)$$  \hspace{1cm} (19)

Panel A of Figure 4 shows the impulse responses of prices, the slope of the futures curves, and storage levels to the different shocks in the model. Generally, any shock that increases the slope of the futures curve will invoke a positive storage response.

Shocks to oil production are generally ameliorated by an opposing storage response, so that a decrease in production leads to a decrease in storage as inventories are used to make up the shortfall. This effect is much stronger in the responsive supply calibration, due to the fact that storage levels depend on future changes in prices, not on the current price. This results in a reduction in overall oil price volatility and a flattening of the term structure of volatility. In the unresponsive regime, even though oil supply shocks are more costly, there is no incentive to release oil in storage, because the lack of oil supply response means there is a much smaller
impact on the slope of the futures curve. These effects are consistent with those described by Dvir and Rogoff (2009).

Panel B of Figure 4 shows a sample simulation path from the responsive regime in the model, and shows that storage closely follows the slope of the futures curve as it does in the data.

D.3 Model Futures Curves with Storage and Stochastic Oil Supply Volatility

The addition of storage and oil supply volatility have little impact on the macroeconomic implications of the model, so I do not report the full calibration results. The primary impact of both changes is seen in the term structure of oil futures across the responsive and unresponsive calibrations. These are reported in Figure 5.

As the figure shows, including stochastic volatility has little effect on the slopes of the futures curve. Including storage in the model mutes the changes in the slope of the futures curve, as storage responds to offset short-term productivity shocks. As discussed previously, this is more pronounced in the responsive regime, and hence results in a flattening of the term structure of futures prices. The change in the slope of the futures curve is largely unaffected by this addition.

Appendix E: Correlation of Oil Prices and Equity Prices

As is shown Table 5 in the main paper, the correlation of oil prices and equity prices in the second period is quite strong. This is driven by the very large increase in correlation following the financial crisis. To illustrate that this increase in correlation is not a feature of the data prior to the crisis Figure 6 plots rolling regression coefficients of daily returns to the aggregate stock market over the previous 6 months on daily changes in oil prices. As the figure clearly shows, the onset of the increased correlation was coincident with the financial crisis, and was not a feature of the pre-crisis data, despite the fact that this change is often cited as an impact of financialization (Buyuksahin and Robe (2011)). In fact, for the year prior to the crisis, the correlation between oil prices and equities was significantly negative, consistent with the prediction of the model.

References


Christoffersen, Peter, and Xuhui Nick Pan, 2014, Oil Risk Exposure and Expected Stock Returns, *Available at SSRN 2399677*.


Table 1: Moments: Benchmark and Alternate Model Specifications

This table lists unconditional moments for aggregate oil specific variables from six different parameterizations of the model. The unresponsive and responsive regimes differ in their values of \( \rho_w \) as described in Table ??.

For the “No Exogenous TFP Effect” specification the parameterizations are the same as the benchmark except that the variable \( \zeta = 0 \). For the “Low Oil Expenditure” calibration the mean level of the oil endowment \( \bar{w} \) is higher than the benchmark specification so that the total average expenditure on oil is roughly half that of the benchmark calibration. The model is simulated for 100 simulations of 480 months, and moments are calculated as the average means or standard deviations of the last 360 months of each simulation.

### Panel A: Aggregate Moments

<table>
<thead>
<tr>
<th>Variables</th>
<th>Benchmark Model</th>
<th>No Exogenous TFP Effect</th>
<th>Low Oil Expenditure</th>
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<tr>
<td>Macroeconomic Quantities</td>
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</tr>
<tr>
<td>( E[\Delta y] )</td>
<td>1.80</td>
<td>1.80</td>
<td>1.80</td>
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<tr>
<td>( \sigma(\Delta y) )</td>
<td>2.35</td>
<td>2.37</td>
<td>2.37</td>
</tr>
<tr>
<td>( \sigma(\bar{y}) )</td>
<td>1.29</td>
<td>1.20</td>
<td>1.35</td>
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<tr>
<td>( E[f/y] )</td>
<td>7.89</td>
<td>7.70</td>
<td>8.68</td>
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<tr>
<td></td>
<td>28.14</td>
<td>27.00</td>
<td>28.32</td>
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<td>Stock Market and Risk Free Rate</td>
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<tr>
<td>( E[r_f] )</td>
<td>1.18</td>
<td>1.56</td>
<td>0.88</td>
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<tr>
<td>( \sigma(r_f) )</td>
<td>0.50</td>
<td>0.50</td>
<td>0.51</td>
</tr>
<tr>
<td>( E[r_{LEV}^{ex}] )</td>
<td>6.71</td>
<td>5.25</td>
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<td>( \sigma(r_{LEV}^{ex}) )</td>
<td>7.12</td>
<td>7.09</td>
<td>7.12</td>
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### Panel B: Oil Price Moments

<table>
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<th>Variables</th>
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<th>Low Oil Expenditure</th>
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</thead>
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<td>Oil Expenditure Ratios</td>
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<tr>
<td>( E[(G+O)P] )</td>
<td>3.68</td>
<td>3.60</td>
<td>3.81</td>
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<tr>
<td>( E[(G+O)P] / Y]</td>
<td>61.68</td>
<td>61.81</td>
<td>61.73</td>
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<td>Oil Futures Prices and Returns</td>
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<tr>
<td>( E[f_2 - f_1] )</td>
<td>2.20</td>
<td>10.42</td>
<td>0.17</td>
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<tr>
<td>( E[r^2 - \bar{r}] )</td>
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<tr>
<td>( \sigma[r_{12}] )</td>
<td>33.64</td>
<td>33.60</td>
<td>33.87</td>
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<tr>
<td>( \sigma[r_{12}] )</td>
<td>17.37</td>
<td>28.83</td>
<td>17.98</td>
</tr>
</tbody>
</table>

### Regression of Aggregate Market on Oil Price Change

\[
\begin{align*}
\beta_{Mkt}^{\Delta y} & = 0.00 - 0.01 - 0.01 - 0.00 - 0.01 - 0.00 \\
\frac{h^2}{R^2} & = 0.00 - 0.00 - 0.00 - 0.00 - 0.00 - 0.00 
\end{align*}
\]
Table 2: Changes in Expected Variance Risk Premia

This table reports expected variance risk premia for oil and copper futures over two subperiods, 1997 - 2004 and 2005 - 2012. The log expected variance risk premium is the difference in logs between the price of a synthetic variance swap and the expected realized variance, which is constructed using an ARMA(1,1) specification of realized variance at monthly frequencies. Realized variance each month is calculated as the variance of daily returns to oil futures. The price of a synthetic variance swap is constructed following Trolle and Schwartz (2010). The table reports means for each subperiod and standard errors, as well as the t-stat and p-value of the difference between the two periods. All standard errors are calculated using Newey-West errors with 6 lags.

<table>
<thead>
<tr>
<th></th>
<th>Oil</th>
<th>Copper</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>E(VRP)</td>
<td>0.189**</td>
<td>0.302**</td>
<td>0.438**</td>
<td>0.324**</td>
</tr>
<tr>
<td>SE</td>
<td>(0.054)</td>
<td>(0.034)</td>
<td>(0.068)</td>
<td>(0.058)</td>
</tr>
<tr>
<td>T-stat for Difference</td>
<td>1.80</td>
<td>-1.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P-Value</td>
<td>7.4%</td>
<td>16.3%</td>
<td></td>
<td></td>
</tr>
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</table>
Figure 1: Oil Futures: Benchmark and Alternate Model Specifications

Average future prices and return volatilities of six different parameterizations of the model. The unresponsive and responsive regimes differ in their values of $\rho_w$ as described in Table ???. For the “No Exogenous TFP Effect” specification the parameterizations are the same as the benchmark except that the variable $\zeta = 0$. For the “Low Oil Expenditure” calibration the mean level of the oil endowment $\bar{w}$ is lower than the benchmark specification so that the total average expenditure on oil is roughly half that of the benchmark calibration. The model is simulated for 100 simulations of 480 months, and moments are calculated as the average means or standard deviations of the last 360 months of each simulation. Future prices are shown in logs and normalized so $E[f^1] = 0$. Future returns averages and standard deviations are monthly.
Figure 2: Model Impulse Responses: Oil Volatility shock

Response of model variables to one standard deviation shock to the volatility of the oil supply. Results are shown for the responsive and unresponsive cases of the Benchmark Model described in Table 4 in the main text.
Figure 3: Oil Storage and the Term Structure of Oil Futures

This figure plots the slope of the futures curve, calculated as the log-difference of oil futures prices with 12-months and 1-month to maturity, along with oil inventories in millions of barrels. Inventory data are Crude Oil Inventories excluding the Strategic Petroleum Reserve from the EIA.
Figure 4: Model Storage Dynamics

Panel A shows response of model storage and futures prices to one standard deviation shocks short and long run productivity shocks as well as shocks to the oil supply. Results are shown for the responsive and unresponsive cases of the Benchmark Model described in Table 4 in the main text. Panel B shows the log of the level of storage relative to the long-run mean as well as the slope of the futures curve calculated as the difference between the 12- and 1-month future contract for a single simulation sample path of the Benchmark model under the responsive oil supply regime. For both panels the slope of the futures curve is the difference between the log of the 12- and 1-month futures prices.
Figure 5: Oil Futures: Benchmark and Alternate Model Specifications

The model is simulated for 100 simulations of 480 months, and moments are calculated as the average means or standard deviations of the last 360 months of each simulation. Future prices are shown in logs and normalized so $E[f^t] = 0$. Future returns averages and standard deviations are monthly.
Figure 6: Aggregate Market Returns and Oil Price Changes

The figure shows the estimated regression slope and 95% confidence interval for rolling 6-month regressions of daily aggregate stock market returns on daily changes in oil prices. Stock returns are from CRSP and spot prices are the WTI index.