Oil Prices and Long-Run Risk*

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ABSTRACT

I add an oil good endowment to the Long-Run Risk model of Bansal and Yaron (2004) to study the asset pricing implications of a constrained oil supply. Lack of responsiveness of the oil endowment changes both the physical and risk-neutral dynamics of oil prices, and explains significant differences in the observed behavior of oil futures prices and returns from 2004 to 2008 relative to the prior 15 years. The model predicts that an unresponsive oil supply increases the risk of exogenous oil shocks, but mitigates risk from other shocks to growth, thereby lowering overall economic risk and the equity premium.

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The importance of oil as an input into the macroeconomy, and its ability to predict future economic growth, suggests that the oil price is an important variable to consider in the context of consumption based asset pricing models.\textsuperscript{1} Though these models have had substantial success in linking exposure to macroeconomic risk to the observed behavior of equity prices, there has been little work examining oil price risk in this context.

This paper presents a new empirical fact; real oil prices can be closely approximated by a function of the relative levels of household oil good consumption (ie. Gasoline) and consumption of other goods. This finding allows for an expression of the oil price in an endowment model of consumption, where the price is implied by the optimizing behavior of a representative consumer. Motivated by this result I add an oil good endowment the Long-Run Risks (LRR) framework of Bansal and Yaron (2004).

The model is employed to study how changes in the endowment process of an oil good affect the behavior of spot prices and oil futures prices, as well as the importance of oil as a risk factor in the economy. The model shows that an oil supply which is unresponsive to prices can explain striking changes in the dynamics of spot and futures prices in recent years. Additionally, the model suggests that these changes have important implications for the overall structure of risk in the economy.

Following 15 years of highly stable oil prices, the period from the end of 2003 to the middle 2008 was characterized by a dramatic rise in oil prices and intense worries about the available supply of oil.\textsuperscript{2} Although oil prices have fallen in recent years, this time period remains an interesting one to study, because it provides a glimpse into the potential impacts of scarce oil and high oil prices. Though not as publicized as the rise in the level of spot prices, several other striking changes in the behavior of spot and futures prices occurred over this time period. These can be summarized as i) A decrease in the mean-reversion of oil prices ii) A switch of term structure of oil futures prices from backwardation to contango and iii) A change in the slope of the term structure of futures returns, from downward sloping to upward sloping. The changes in the term structure of futures are illustrated by

\textsuperscript{1}Hamilton (2005) documents that oil shocks have a significant negative relation with future GDP growth for 1973 - 2005. I confirm this relationship holds for aggregate consumption as well.

\textsuperscript{2}The specific choice of this period is motivated by formal structural break tests discussed in Section 3.
Figure 1: The Term Structure of Crude Oil Futures

Panel A reports the average log of futures prices for the two halves of the sample. Both curves are normalized so that $f^3 = 1$. Panel B reports the averages of monthly returns for futures prices. Returns are the average return in excess of the return on the two month future. Panel C reports monthly volatility of oil returns. Data for NYMEX futures prices on Crude Light Sweet Oil of up to 12 months to maturity.
The model shows that these changes in price dynamics can be explained by a change in the dynamics of the oil supply, represented in the model by a change in the parameters which govern the oil consumption good endowment. Specifically, a reduction in the ability of the oil supply to respond to changes in oil prices can explain the significant observed decrease in the mean-reversion of spot prices over the 2004 to 2008 period. More importantly, it also generates significant changes in the risk premia associated with oil prices, which in turn create changes in the behavior of oil futures. The change in riskiness implied by the changes in endowment dynamics is able to explain the observed behavior of the futures curves.

To interpret the changes in the context of the model, it is useful to consider what mean-reverting oil prices imply in the context of consumption. In the consumption model, times of high oil prices are times with a relatively scarce endowment of oil. If the oil supply is very responsive to prices, high oil prices are quickly followed by rising oil consumption in future periods and correspondingly a drop in prices. If oil consumption growth no longer exhibits this response, high oil prices persist and therefore oil prices exhibit significantly less mean-reversion. Equivalently, shocks to oil prices become more persistent.

This increase in persistence also creates a change in the price of risk, which shifts the futures price curve from backwardation to contango. In the LRR framework, the persistence of growth shocks is the main driver of their corresponding price of risk, and therefore oil shocks command a larger risk premia in the recent period. This risk premia is negative since high oil prices predict low future growth, and therefore imparts an upward slope on the term structure of oil prices, consistent with the change from backwardation to contango observed in the data.

Another similar effect generates the change in the term structure of futures returns. In the second period an unresponsive oil supply implies that changes in expectations of future aggregate consumption growth will have a larger effect on expectations of future oil prices. If expected aggregate consumption growth is high in a time of highly responsive oil supply, oil consumption is able to rise in the future with aggregate consumption so

\[^3\text{From the perspective of an endowment economy oil becomes less scarce in future periods as the supply increases.}\]
that there is no expected change in oil prices. Conversely, in a more constrained period, oil consumption growth does not respond to changes in expected aggregate growth, and thus high expected growth results in high expectations of future oil prices. Since long-term futures load more heavily on these changes in expected growth that short-term futures, this generates a positive risk premia for long-term futures prices, generating the upward slope in the term structure of returns.

The two effects also have intuitive, yet opposite effects on the overall level of risk in the economy. The increased risk associated with oil price shocks leads to an increase in overall economic risk. However, the high (low) expectations of future oil prices that accompany increases (decreases) in expected aggregate growth serve to mitigate the effect of these growth shocks, which serves to reduce overall risk. This effect, that oil prices act as a counterweight to changes in aggregate economic growth is one that has been central to discussions surrounding the financial crisis and the subsequent recovery. One of the few silver linings of the period following the ”Great Recession” of 2008 and 2009 was the significant reduction in oil prices created by lower demand. As the economy begins to grow again, there is concern that the high oil prices that come along with increasing demand have the potential to slow down the recovery. Though the intuition is not novel, the contribution here is to show that this effect is particularly strong in a LRR setting due to the highly persistent nature of the long-run aggregate growth shocks. In fact, in my calibrations I find that it is this second effect which dominates, and the total effect is a significant reduction the overall equity premium.

The results here mapping changes in consumption dynamics into a change in the riskiness of oil futures are important not only because they give insight into the behavior of oil prices, but also because they provide evidence for two very important aspects of the general LRR model: (i) The relation between the persistence of shocks to growth and their associated level of risk, and (ii) the relation between the timing of cash flows and their associated risk premium. These two effects are difficult to observe in the standard consumption and equity data, since expectations of future consumption growth and the persistence of shocks to this growth are generally difficult to identify, and since the risk associated with specific
dividend payments at different horizons is hard to identify from equity prices. Intriguingly, the upward sloping term structure of returns in oil futures provides potential evidence supporting the high-riskiness of long term cashflows, which is an important feature of LLR models as discussed by Hansen, Heaton, and Li (2008).

Since oil prices predict future consumption growth, the existence of oil futures contracts makes oil prices an ideal laboratory in which to study these effects. Comovement of futures contracts with the spot price allows for much more accurate measurements of expectations of persistence than a single time series (see Bessembinder, Coughenour, Seguin, and Smoller (1995)), a fact which I utilize to identify a change in expected persistence over a very short time period. Additionally, the cross-section of different maturities allows for observation of risk premia at different time horizons. Since returns of futures contracts with different maturities are highly correlated, the differences in returns of futures contracts with different horizons is much less volatile than the returns on a single contract. I use this feature of the data to identify significant differences in average returns at different maturities over my very short sample. To my knowledge this is a novel approach and may have uses in other contexts.

The split date for the sample is selected using a formal test for a structural break in the mean-reversion of oil prices, and yields an optimal break date in October of 2003. I have data on oil futures for 1987 - 2010, but I only consider the time period prior to the financial crisis. This avoids concerns of results driven by the large movements in oil prices during and after the crisis, but including this period does not significantly affect the results. Though the model here is primarily focused on consumption dynamics and prices, it is interesting to note that this period is one for which data on oil production seems to indicate that the oil industry was constrained by available production capacity. Figure 2 illustrates that the available capacity of the OPEC countries (the only countries for which these data are available) was clearly in a low state during the period from 2003 until the financial crisis of 2008, precisely when the effects observed in asset prices were the strongest. Also during

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4Binsbergen, Brandt, and Koijen (2010) construct synthetic dividend strips from option values to study the risk associated with individual cashflows. They find that risk premia do not increase as the time to realization of the cashflow increases, and interpret this as evidence against the long-run risk formulation.
this period, world oil production showed little growth despite a dramatic rise in oil prices. Although the model here is an endowment economy and therefore abstracts away from production in order to focus on asset pricing implications, the inability of oil consumption (the supply of oil in the endowment economy) to respond to changes in prices is perfectly consistent with the observed state of oil production.

**Figure 2: World Oil Production and OPEC Spare Production Capacity**
Data for production and OPEC excess capacity from the Energy Information Association.

![Figure 2: World Oil Production and OPEC Spare Production Capacity](image)

Much of the literature on commodities prices has its roots in the theory of storage (Kaldor (1939), Working (1949), Telser (1958)) and until very recently, most work in this area fell into one of two categories. The first specifies an exogenous process for the stock price to examine the pricing implications for derivative contracts (Brennan and Schwartz (1985), Gibson and Schwartz (1990), Schwartz (1997)), while the second uses the theory of
storage to derive implications of the price of oil (Williams and Wright (1991), Deaton and Laroque (1992), Deaton and Laroque (1996), Routledge, Seppi, and Spatt (2000)).

Though an endowment model of consumption, the model here is similar to the reduced form models in the sense that it can be recast as a joint model of the spot price, the convenience yield, and the risk-free rate, with each variable exposed to exogenous shocks. The important difference is that the shocks in this model represent shocks to the underlying consumption process, and therefore the risk-premia are functions of the parameters of the endowment process and the utility specification, rather than exogenous inputs themselves. However these constraints also mean that the endowment model, while providing an excellent qualitative match for the changes in the behavior of futures prices, on some dimensions provides a poor fit for the unconditional shape of the futures curve. Therefore, this model should not be viewed as a replacement for reduced form models, but rather a way to shed some light on some of the fundamental forces driving futures prices and oil price risk.

More recent research (Carlson, Khokher, and Titman (2007), Kogan, Livdan, and Yaron (2009)) has focused on oil production to generate futures price dynamics. These recent studies focus primarily on dynamics of the futures prices under the physical measure, and while they allow for a specification of the risk premium, they do not provide a theoretical explanation of the price of commodities risk. Casassus, Collin-Dufresne, and Routledge (2005) develop a general equilibrium model with oil as an input into the production of a single consumption good, and study the implications of oil price risk in this context. They also find that oil price risk can change based on the condition of oil production. However, the mechanism relies on the distance of the oil price from the level necessary to induce further investment in oil wells, and is therefore distinct from the effects described here, which reflect a more fundamental shift in the dynamics of oil consumption.

Studies applying traditional asset pricing models to explain risk premia in commodity prices have met with limited success (see Dusak (1973), Breeden (1980) and Jagannathan (1985)). Another common theory to explain the observed positive risk premia, or "Normal Backwardation" as introduced by Keynes (1930), postulates that producers who are seeking to hedge risks of future price movements are willing to pay a premium to speculators.
Gorton, Hayashi, and Rouwenhorst (2007a) show that Sharpe Ratios of commodities prices over the last 40 years are significantly higher for commodities futures than for equities, and that levels of inventory predict futures returns, which they interpret as support for this theory. While the results here may help shed some light on the source of risk premia in commodities, it is important to keep in mind that the results in this paper depend greatly on the relations between oil prices and consumption which are unique among commodities.

The rest of the paper is organized as follows. Section 1 describes the basic model and discusses how changes in parameters governing the responsiveness of oil consumption create changes in risk. Section 2 describes the observed behavior of consumption and oil prices, and documents the changes in these dynamics as well as the changes in the term structure of oil futures prices over the sample period. Section 3 discusses extensions to the model and calibrates the model to match salient moments of asset prices and consumption. Section 4 concludes.

I The Basic Model

In this section I consider a simplified version of the model and derive closed form approximate solutions. The model adds an oil consumption good to the long run-risk framework. Recent work by Yang (2010) emphasizes that durable consumption growth exhibits much higher persistence than nondurable consumption growth, and that this higher persistence can be used in a model of long-run risk to explain the equity premium and risk-free rate puzzles. I find that this higher persistence is important in explaining the observed term structure of oil futures. I also find that including durable goods strengthens the relation between levels of consumption and the spot price of oil.

Considering durable consumption and nondurable consumption separately generates an extra term in the stochastic discount factor when using Epstein-Zin Preferences, reflecting the fact that consuming a durable good exposes the representative agent to price risk generated by the changing composition of consumption\(^5\). I assume that \(C_t = N_t^{1-\alpha} D_t^\alpha\),

\(^5\)For a full discussion of the issues involved using durable consumption in a model with Epstein-Zin preferences
where \(N_t\) is the expenditure on nondurables and services excluding oil, and \(D_t\) is the services flow from the stock of consumer durable goods, which is assumed to be linear in the stock. I consider this aggregation as the consumption good, and all prices in the model will be in terms of units of this good.

Pakos (2004), considers a model with utility arising from an aggregation of nondurable and durable goods using a Generalized Constant Elasticity of Substitution (GCES) felicity function. Here I follow Yang (2010) and consider a Cobb-Douglas aggregate of durable and nondurable goods, I then use the GCES functional form to represent utility across the aggregate consumption good, \(C_t\), and an oil consumption good, \(O_t\). The representative consumer has utility \(V_t(C_t, O_t)\) in each period, where

\[
V_t(C_t, O_t) = \left[ (1 - a)C_t^{1 - \frac{1}{\rho}} + aO_t^{1 - \frac{\eta}{\rho}} \right]^{\frac{\rho}{\rho - 1}} \tag{1}
\]

This function nests several of the commonly used utility functions. For \(\eta = 1\), \(V_t\) is the standard Constant Elasticity of Substitution (CES) function. For \(\rho = 0\) the function is the Leontieff function and for \(\rho = 1\) the function is Cobb - Douglass. I find empirically that \(\rho < 1\) suggesting that oil consumption is a complement to aggregate consumption rather than a substitute, and that \(\eta\) is substantially greater than one, suggesting that oil consumption goods are necessary, rather than luxury, goods. Given this function, optimal behavior by the consumer implies that the price of oil in terms of units of the aggregate consumption good is the ratio of the marginal utilities of oil and aggregate consumption.

\[
P_t = \frac{a(1 - \frac{\eta}{\rho})C_t^{\frac{1}{\rho}}}{(1 - a)(1 - \frac{1}{\rho})O_t^{\frac{\eta}{\rho}}} \tag{2}
\]

Taking logarithms, where \(p_t, c_t, o_t\) representing logs of price, aggregate consumption and oil consumption, yields

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see Yogo (2005) and Yang (2010)
\[ p_t = \text{constant} + \frac{1}{\rho}(c_t - \eta o_t) \] (3)

Dealing with non-homothetic preferences complicates the model along several dimensions. For instance, if prices are stationary, then the ratio of expenditure on oil consumption to expenditure on basic consumption \( \left( H_t = \frac{c_t}{o_t} \right) \) will be non-stationary, as it will be a function of the price and the absolute level of consumption. In order to solve this issue, in the model the endowment will be specified so that \( H_t \) is stationary to keep the problem well defined.

This intratemporal utility function is embedded within Epstein and Zin (1989) preferences so that total utility is

\[ U_t = \left[ (1 - \delta) V_t^{1-\gamma} + \delta \left( E_t[U^1_{t+1}] \right)^{1/\beta} \right]^{1/\gamma} \] (4)

Where \( \gamma \) is the coefficient of risk aversion and \( \psi \) is the intertemporal elasticity of substitution (IES). Having specified the utility of the representative agent, what is left is to specify dynamics of oil consumption and aggregate consumption. The consumption dynamics have the following form.

\[
\begin{align*}
\Delta c_{t+1} &= \mu_c + \pi_c \left[ h_t - \bar{h} \right] + x_t + \sigma_{c,t} \epsilon_{t+1}^c \\
\Delta o_{t+1} &= \mu_o + \pi_o \left[ h_t - \bar{h} \right] + \Phi_x x_t + \sigma_o \epsilon_{t+1}^o \\
x_{t+1} &= \rho x_t + \varphi_x \sigma_{c,t} \epsilon_{t+1}^x \\
h_t &= \frac{1}{\rho} (c_t - \eta o_t) + o_t - c_t \\
\sigma_{c,t+1} &= \nu \left( \sigma_{c,t}^2 - \sigma_{c,0}^2 \right) + \sigma_{c,0}^2 + \sigma_w \epsilon_{t+1}^w \\
\Delta y_{t+1} &= \mu_y + \chi E_t[\Delta c_{t+1}] + \varphi_y \sigma_{c,t} \epsilon_{t+1}^y
\end{align*}
\] (5)
Here $o_t$ represents log of oil consumption, $c_t$ is log of aggregate consumption, $x_t$ is a predictable component in long run aggregate consumption growth, and $y_t$ is the log of the aggregate dividend. Given the expression for oil prices from Equation 3, $h_t$ is the log of the expenditure ratio. Including $h_t - \bar{h}$ as an error correction term ensures that the time series of $h_t$ is $I(0)$. Moreover, this term generates a predictive relation between high oil prices and future growth, since $h_t$ is high when oil prices are high. With a negative value of $\pi_c$, high oil prices will predict low future growth in aggregate consumption, $c_t$. Notice if $\eta = 1$, the intratemporal utility is CES and hence $h_t = (1 - \rho)p_t$ and the model implies a stationary price. However if $\eta > 1$, as is the case in the data, the ratio of expenditure will remain stationary but there will be an upward drift in the price of oil.

This specification combines features of both Bansal and Yaron (2004) in that it includes a separate process for the predictable consumption rate growth as well as time-varying volatility of aggregate consumption growth, and Hansen, Heaton, and Li (2008) in that it includes an additional source of predictable consumption growth coming from the error correction term ($h_t - \bar{h}$). Dividends can be interpreted a levered claim on consumption, as in Bansal and Yaron (2004), and have expected growth which is perfectly correlated with expected growth to the aggregate consumption endowment. Correlation among the innovations is straightforward to include, but for parsimony here I assume they are independent of each other, and i.i.d. with a $N(0,1)$ distribution.

The shock to $e_t^o$ represents an innovation to oil consumption, which is also an innovation to the oil price which is unrelated to a change in $c_t$. It is important to note that a positive innovation to $e_t^o$ represents a negative innovation to $p_t$, so that as oil becomes less scarce the price falls.

The $x_t$ component represents a predictable component of consumption growth similar to the model of Bansal and Yaron (2004). This model is sometimes criticized for the low level of predictability in consumption growth. However, as Yang (2010) shows, there is in fact significant predictability in durable consumption growth. This predictability is also present in the Cobb-Douglas aggregation of durable and nondurable consumption used here.
In addition to $x_t$, there is also predictable growth coming from the error correction term $(h_t - \bar{h})$. In this sense this model is similar to that of Hansen, Heaton, and Li (2008), which specifies a similar cointegrating relation between dividends and earnings, analogous to the relation in this model of basic consumption and oil consumption. It is important to note here that this specification implies that the ratio of expenditure on oil consumption to aggregate consumption is an $I(0)$ variable. Equivalently, it implies a cointegrating relation between $c_t$ and $o_t$, and two cointegrating relations between $c_t$, $o_t$, and $p_t$. I find support for this specification from Johansen (1991) tests and estimates of a vector error correction model of oil consumption and aggregate consumption. These results are reported in Appendix B.

This model here is a slightly simplified version of the model I take to the data. I make one addition to the utility function and add an external habit to the specification for oil prices. This allows for a better quantitative fit of futures curves and price volatilities but does not in any way effect the qualitative implications of the model. This extension is discussed in more detail in Section A.

### A Model Solution

I solve for approximate log-linear solutions in the manner of Bansal and Yaron (2004), with an extra approximation necessary to deal with the GCES intratemporal utility, namely that $V_t = C_t$. 

This approximation is not imposed in calculating the numerical results though it does not significantly alter the results. This fact highlights an important distinction between this model and the models of Pakos [2004], Yogo [2006], and Yang [2010], which rely on the degree of substitution between durable and nondurable consumption to generate asset pricing implications. Here the implications of the model are not driven directly by the utility derived from oil consumption, but rather by the impact of oil prices on predicting

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6This is appropriate due to the small observed value of expenditure on oil consumption to expenditure on basic consumption goods. The intratemporal utility can be rewritten as $C_t \left( (1 + \rho)(1 + \frac{1}{\bar{H_t}}) \right)^{\frac{1}{\rho}}$. Details are in Appendix A
future consumption growth.

The solution proceeds by first deriving an expression for the stochastic discount factor. Section B derives expressions for futures prices and their loadings. Section D provides intuition for how changing two parameters, $\Phi_x$ and $\pi_o$, in the consumption process changes both the prices of risk and the loadings of futures to generate observed changes in the term structure.

The price-dividend ratio for the consumption claim is linear in two states, $x_t$ and $p_t - \bar{p}$

$$z_t = A_0 + A_1 x_t + A_2 (p_t - \bar{p})$$  

(6)

Exploiting the pricing equation

$$1 = E_t [\exp (m_{t+1} + r_{g,t+1})]$$  

(7)

Allows for solution of the coefficients. The coefficients for $A_1$ and $A_2$ are given by

$$A_1 = \frac{(1 - \frac{1}{\psi}) + A_2 \kappa_1 (1 - \eta \Phi_x)}{1 - \kappa_1 \rho_x}$$  

(8)

$$A_2 = \pi_c \frac{(1 - \frac{1}{\psi})}{1 - \kappa_1 (1 + \pi_c - \pi_o)}$$  

(9)

These values are very similar in flavor to the coefficient for the long-run risk shock, $x_t$ in the standard formulation of Bansal and Yaron (2004). The expression $A_2$ takes the sign of $\pi_c$, and represents the contribution of the predictable growth in consumption generated by the oil price to the expected consumption to wealth ratio. The $A_1$ term is the same as

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7Since preferences are homothetic ($\eta = 1$), $h_t$ is a multiple of $p_t$ and therefore $p_t - \bar{p}$ is substituted for the endowment error-correction term.

8I ignore the contribution of stochastic volatility here since in my calibrations it has a small effect. Solutions in Appendix A as well as the final numerical solutions include this effect.
that of Bansal and Yaron (2004) with an additional term generated by the effect of \( x_t \) on the oil price. These values can then be used to calculate the log of the pricing kernel, with the innovation having the following form.

\[
m_{t+1} - E_t[m_{t+1}] = -\lambda_c \sigma_{c,t} \epsilon_{t+1}^c - \lambda_o \sigma_{o,t} \epsilon_{t+1}^o - \lambda_x \varphi_x \sigma_{c,t} \epsilon_{t+1}^x
\]

and the prices of risk associated with each shock are given by

\[
\lambda_c = \gamma + (1 - \Theta) A_2 \kappa_1 \\
\lambda_o = -(1 - \Theta) A_2 \kappa_1 \\
\lambda_x = (1 - \Theta) A_1 \kappa_1
\]

The first term in Equation (11) is the standard Breeden [1980] CCAPM term adjusted to account for the rise in oil prices created by a rise in basic consumption. The second term represents innovation due to shocks to oil consumption, or equivalently oil price shocks. The third is the innovation to long run expectations in consumption growth as in Bansal and Yaron (2004).

**B Oil Futures Prices**

The oil futures price\(^9\) for a future with maturity \( j \) is described by the equation

\[
0 = E_t [ M_{t+1} (F_{t+1}^j - F_t^j) ]
\]

Exploiting the log-normality of both \( P_t \) and \( M_t \) and rearranging yields the following expression for the log of futures prices.

\(^9\)I assume that futures are marked to market on a monthly basis
\[ f_t^j = E_t[f_{t+1}^{j-1}] + \frac{1}{2} \text{var}_t(f_{t+1}^{j-1}) + \text{cov}_t(f_{t+1}^{j-1}, m_{t+1}) \] (16)

That is the futures price is the log of the expected futures price for the same maturity one month from now, plus a covariance term that reflects the riskiness of the contract. While closed form expressions for various futures contracts are messy, they can be calculated through a simple recursive algorithm.

Futures prices can be expressed as linear function of the state variables

\[ f_t^j = B_0^j + B_x^j x_t + B_p^j (p_t - \bar{p}) + B_p^j \sigma_{c,t}^2 \] (17)

Where the expressions are given in Appendix B. The initial value of the recursion represents the relation \( f_t^0 = p_t \), so \( B_0^p = 1 \), \( B_0^x = \bar{p} \), and \( B_0^x = 0 \).

These equations can also be used to calculate the expected returns on a futures contract. The expected return is

\[ E[r_t^j] = E_t[f_{t+1}^{j-1} - f_t^j] + \frac{1}{2} \text{var}_t(f_{t+1}^{j-1}) \] (18)

The expected returns on futures depend on the loadings of futures prices on the three state variables that describe the stochastic discount factor and the prices of risk of shocks. In the full model there are four shocks with associated prices of risk, the three in Equation 11 as well as shocks to stochastic volatility. However, as mentioned previously, the shocks to the stochastic volatility component do not have a significant price of risk associated with them. Therefore expected return can be written as

\[ E[r_{t+1}^j] \approx \frac{1}{\rho} B_p^j (\lambda_c \sigma_{c,t}^2 - \lambda_o \sigma_{o,t}^2) + B_p^j \lambda_x \varphi_x \sigma_{c,t}^2 \] (19)

The long-run risk framework generates intuitive linear factor model to explain the expected returns on oil prices. The return of future \( j \) depends on its loading on the two state
variables (the $B$ terms) and the prices of risk associated with exposure to each shock (the $\lambda$ terms). Positive risk premia come from exposure to the shocks to short-term consumption growth as well as the long-run growth $x_t$, while negative risk premia are generated by exposure to shocks to oil consumption. The two parameters governing oil supply response, $\pi_o$ and $\Phi_x$, have implications for both the loadings and prices of risk, and therefore change the expected return on futures prices.

C Model Implied Convenience Yields and Spot Prices

Much of the prior work on oil prices follows the work of Gibson and Schwartz (1990) and jointly specifies reduced form processes for the spot price, the interest rate, and a "Convenience Yield", which describes the benefit that flows to a holder of the physical asset net of storage costs. Here I consider the discrete-time log-linear approximation of this set up, where the convenience yield $\delta_t$ is defined using the slope of the futures curve

$$f_{1,t} - p_t = r_{f,t} - \delta_t - \frac{1}{2} \text{var}(\Delta p_{t+1})$$

(20)

Therefore the risk-free rate, the convenience yield, and the spot price can be thought of as being jointly described by three factors, corresponding to the three states of Equation (17), $p_t - \bar{p}$, $x_t$, and $\sigma_{c,t}^2$. Given that the spot price is

$$p_t = \frac{1}{\rho} (c_t - o_t)$$

The risk free rate can be described as begin linear in the spot price and $x_t$ so that

$$r_{f,t} = B^r_0 + \frac{1}{\psi} x_t + \frac{1}{\psi} \pi_c (p_t - \bar{p}) + B^r_\sigma \sigma_{c,t}^2$$

(21)

Where $B^r_0$ and $B^r_\sigma$ are constants that are functions of the consumption and utility parameters.
Finally the convenience yield is given by
\[
\delta_t = B_{0}^f + \lambda_o \frac{1}{\rho} \sigma_o^2 - \lambda_c \frac{1}{\rho} \sigma_c^2 + (1 - \pi_c + \pi_o)(p_t - \bar{p}) + \frac{1}{\rho} (1 - \Phi_x) x_t \tag{22}
\]

This is system is very similar to the model of Casassus and Collin-Dufresne (2005), though not identical. First, the risk free rate is not governed by a single factor with a single mean reversion but rather is a function of all three of the factors. Likewise with the convenience yield. More importantly, when considering risk neutral vs. physical dynamics, the risk-premia in this model are functions of the growth process and utility parameters rather than exogenous inputs. Though the very general model of Casassus and Collin-Dufresne (2005) allows these risk premia to change based on the levels of the state variables, the parameters of the process remain fixed. One could easily calibrate the reduced form models over the two time periods with different parameters and obtain different specification for risk premia to fit the data, however there would be no theoretical underpinning for why these values change. Here I will be focusing on changes in the parameters \(\pi_o\) and \(\Phi_x\), and examining how the changes in these parameters endogenously generate changes in risk premia.

From the expression for the convenience yield, it is easy to see how changes in parameters might affect the risk premia in the consumption model. Changes in \(\pi_o\) will affect the speed of mean reversion, as well as the average slope of the futures curve through a change in the value of \(\lambda_o\). Changes in the value of \(\Phi_x\) will affect the exposure of the convenience yield to the persistent growth component, \(x_t\). Due to the high price of risk associated with this exposure, this will have an impact on the risk associated with futures of longer horizons. The next section explores these effects in greater detail.

D Changing \(\Phi_x\) and \(\pi_o\)

I will be focused on the effects of changing the values of two parameters, \(\pi_o\) and \(\Phi_x\). Explaining the observed behavior of asset prices will motivate these changes, however it is worth discussing what they represent in an economic sense. The advantage of developing an
endowment economy of consumption is that the economist may be agnostic to the sources of the shocks to consumption, while still being able to make inferences about their effect on asset prices. It is important to keep in mind however, that behind this model there is a real economy of production, supply, and demand which is generating the observed dynamics in consumption. I view the changes in parameters as a reasonable reflection of changes to the state of oil production.

In a state of the world where production capacity is plentiful, given a shock to oil prices, say from a disruption of a major producer, it is reasonable to expect that production from other sources would quickly rise to bring prices back to a stable long-run mean. However, given a state where production capacity is scarce, a shock to a single source could have long run impacts. This change would be reflected in an endowment economy by a reduction in the value of $\pi_o$.

Additionally, given plentiful excess capacity, a state with high expected aggregate growth would not necessarily imply a rise in future prices, since oil production would be expected to rise as well. However, if production is constrained it is reasonable to think that high expectations of future aggregate growth might lead to rising oil prices as production is unable to respond. This is captured in the endowment by a reduction in the value of $\Phi_x$.

Both of these changes in parameters can be explained by an inability of the oil industry to increase supply in response to changes in demand over the second half of the sample. Indeed, as Figure 2 shows, the period for which changes in oil price behavior are most striking, from the middle of 2003 up until the beginning of the crisis in 2008, was characterized by an oil industry with very little excess production capacity and low overall growth in oil production.

For the value of $\Phi_x$, a VECM model for the consumption data prior to 2003 suggests a positive value, consistent with expected oil consumption increasing in times of positive expected aggregate consumption growth. In fact, it is high enough to imply that expected growth in consumption implies negative growth in oil prices. This result seems economically unlikely, so therefore I consider $\Phi_x = 1$ in the first period so that a shock to $x_t$ has no
effect on future oil prices. For the second period the short data period (5 years) makes it
difficult to obtain accurate estimates with quarterly consumption data. Therefore I appeal
to both the behavior of asset prices and the general intuition regarding an unresponsive oil
production sector and set $\Phi_x = 0$. With this value an increase in $x_t$ has no effect on the
future supply of the oil consumption good, and hence implies expected future growth in oil
prices.

The parameter $\pi_o$ governs the rate with which oil consumption responds to a change
in price to return prices to the long run stable oil price. The persistence of oil prices is
simply the persistence of the cointegrating vector, $h_t - \bar{h}$, and has a value of $(1 + \pi_c - \pi_o)$. Therefore, a high value of $\pi_o$ will lead to low persistence of oil prices. Both the estimates
from the quarterly consumption data as well as the behavior of oil prices imply a relatively
large value of $\pi_o$ in the first period reflecting a high degree of mean reversion in oil prices. In
the second period the amount of mean-reversion is greatly reduced and therefore I consider
a value of $\pi_o$ close to zero in the second period. For the choice of the parameter $\pi_c$, I keep
the values the same across the two calibrations of the model and focus on the effects of
changes to $\pi_o$ and $\Phi_x$.

To illustrate how changes to these parameters affect oil prices, Figure 3 shows the
impulse responses to shocks to both oil consumption (an oil price shock), and the parameter
$x_t$ (an expected growth shock) under the two different parameterizations of the model. Plots
(a) and (c) show the impulse response of $c_t$, $o_t$, and $p_t$ to a negative innovation to $\epsilon_t^o$, which
is equivalent to a positive oil price shock. As is evident in the first plot, a larger value of
$\pi_o$ means that the high price will induce growth in oil consumption in prior periods, which
will result in a falling oil price. However, in the second period, the lower value of $\pi_o$ means
that the oil price will remain high, or that the shock to oil prices will be more persistent.

This change in $\pi_o$ also has an effect on the response of $c_t$. Though the value of $\pi_c$ is
equal in the two figures, the continuing high oil price means that in the second period, the
negative growth of oil prices persists longer than in the first period. This has an important
effect on the magnitude of the risk premium associated with oil price shocks, since the
persistence of expected growth is the primary determination of the price of growth risk in
Figure 3: Basic Model - Impulse Response Functions

Impulse response function of logs of aggregate consumption ($c_t$), oil consumption ($o_t$), and the oil price ($p_t = \frac{1}{\rho}(c_t - \eta o_t)$) to innovations to oil consumption and the expected growth of aggregate consumption. Period 1:

$$\pi_o = .1 \text{ and } \Phi_x = \frac{1}{\eta}$$

(a) Negative shock to $e_t^o$

(b) Positive shock to $e_t^e$

Period 2: $\pi_o \approx 0 \text{ and } \Phi_x = 0$

(c) Negative shock to $e_t^o$

(d) Positive shock to $e_t^e$
D.1 Changes in Prices of Risk

In order to examine how changes in parameters affect the prices of risk associated with shocks to future consumption growth, it is worthwhile to look more closely at how the coefficients $A_1$ and $A_2$ relate to the standard coefficient on $x_t$ in the model of Bansal and Yaron (2004). That coefficient is

$$A_{BY}^1 = \frac{(1 - \frac{1}{\psi})}{1 - \kappa_1 \rho_x}$$

When $\psi > 1$, this coefficient is positive. Since $\kappa_1 \approx 1$, with a value of persistence, $\rho_x$, near one, this term can be very large implying a large magnitude for the price of risk of shocks to $x_t$. This coefficient is very similar to the coefficient associated with the loading on the oil price $p_t - \bar{p}$ in the model presented here

$$A_2 = \pi_c \frac{(1 - \frac{1}{\psi})}{1 - \kappa_1 (1 + \pi_c - \pi_o)}$$
Here, the value \((1 + \pi_c - \pi_o)\) is the persistence of the oil price, and \(\pi_c\) is the effect the oil price has on consumption growth. Since \(\pi_c\) is negative, if the oil price is persistent then shocks to oil prices will have a large, negative price of risk associated with them. Therefore, the low value of \(\pi_o\) in the second period creates a higher persistence, which amplifies the price of risk associated with oil shocks. This price of risk for shocks to the oil price is also important in determining the price of risk for shocks to \(x_t\), due to the extra term in the associated coefficient

\[
A_1 = \frac{(1 - \frac{1}{\psi}) + A_2 \kappa_1 (1 - \Phi_x)}{1 - \kappa_1 \rho_x}
\]

If \(\Phi_x = 0\), shocks to \(x_t\) will also be shocks to future growth in oil prices, and if oil prices are persistent \(A_2\) will have a large negative magnitude and the extra term will substantially reduce the price of risk for shocks to \(x_t\). This is an algebraic representation of a very intuitive idea. In a world where oil prices are highly persistent and related to the level of consumption, they can act as a ”counterweight” to shocks to expected growth. If high consumption growth is expected then a rise in oil prices is effected as well, which will reduce overall growth.

This counterweighting effect is also present in the price of short-term consumption risk. Recall that the price of risks associated with shocks to \(c_t\) is

\[
\lambda_c = \gamma + (1 - \Theta)A_2 \kappa_1
\]

The second term is negative due to the negative values of \(\Theta\) and \(A_2\). Therefore, when the oil price is persistent, and the magnitude of \(A_2\) is large, and the risk associated with short term consumption shocks is also reduced by the high oil prices they generate.

These effects demonstrate how unresponsive oil prices can act as a natural hedge in the economy. However, a highly persistent oil price can also generate an increase in risk in the
economy through the increased risk associated with shocks to $\epsilon_t^e$, but in my calibrations I find that the reduction of risk from shocks to $x_t$ and $c_t$ is a stronger effect, and results in reduced systematic risk, a lower equity premium, and higher price-dividend ratios.

D.2 Changes in Loadings for Oil Futures

In order to consider how changes in the consumption parameters affect expected returns on oil futures, we also need to examine how they affect the loadings of oil futures prices on the two shocks. The values of $B^j_x$ and $B^j_p$ are determined by the following recursion.

\[
B^j_x = B^{j-1}_x \rho_x + B^{j-1}_p (1 - \Phi_x)
\]
\[
B^j_p = (1 + \pi_c - \pi_o)B^{j-1}_p
\]

With $B^0_x = 0$, and $B^1_p = 1$. In the first period, with $\Phi_x = 1$ and a large value of $\pi_o$, $B^j_x = 0$ for all maturities and $B^j_p$ decays quickly at higher maturities. In the second period, $B^j_p$ decays more slowly with the higher persistence, and $B^j_x \approx \left(\frac{1}{\rho}\right)^j$. Therefore, exposure to shocks to $x_t$ increases linearly across the futures curve.

In the first period, exposure to shocks to $o_t$ commands very little risk premium, and the futures are not exposed to shocks to $x_t$, so that expected return is approximately.

\[
E[r^{j+1}] \approx (1 - \pi_c + \pi_o)^j \frac{1}{\rho} \lambda c \sigma^2_{c,t}
\] (23)

Therefore, the only risk premium is a positive one from the shocks to $c_t$, which quickly diminishes over longer horizons. This generates the typical downward sloping commodities curve observed in the first part of the sample.

However, in the second period things look very different. In the second period, expected return is approximately.
With the increase in the value of $\lambda_o$, the value of the first term is negative, generating an upward term structure. This negative expected return from the negative exposure to shocks to $o_t$ remains approximately constant across the term structure due to the slow decay of $B^j_p$, and therefore generates an upward sloping term structure of prices.

Meanwhile, the second term, representing the exposure to shocks to $x_t$, generates increasing positive expected returns across the term structure. So that long term futures have higher expected returns, consistent with data.

II Empirical Data for Consumption and Oil Prices

A Data Sources

Quarterly data for consumption come from the National Income and Product Account (NIPA) tables. Much of the analysis relies on a novel measure of oil consumption, the personal consumption of "Gasoline and other Energy Goods" from the NIPA survey. This measure includes personal consumption of both gasoline and fuel oils, though in terms of expenditure over 90% of the total comes from expenditure on "Motor Vehicle Fuels, Lubricants, and Fluids" while the remaining 10% is attributed to "Fuel Oil and Other Fuels". Most importantly, this measure is constructed so as not to include consumption for government and corporate use, or consumption of gasoline for energy generation. In this sense it is different from the measure of "Product Supplied" provided by the Energy Information Administration (EIA), which is the typical measure of oil consumption. I divide my measure of personal oil consumption by the level of the population in order to obtain a measure of per capita consumption, as is consistent with literature.

Since gasoline is by far the most important good in this measure, and I am interested in quantifying the utility of consumption, I also adjust for efficiency gains in the use of
gasoline. I calculate this using data from the Bureau of Transportation Safety for the average miles per gallon of the U.S. passenger car fleet. The relative price implied by the agents utility function is then a price for miles rather than a price for gasoline, so I convert it using the miles per gallon to the implied price for oil. For parsimony throughout the description of the model I refer to oil consumption as direct consumption of oil, but for the empirical work I perform these conversions. There is also the potential issue of changes in the efficiency of converting crude to gasoline, but I observe that the price of gasoline and oil have not deviated substantially over the period, and are nearly identical in their innovations, particularly at quarterly frequency.

In order to compare the relative levels of personal consumption of oil to total economic consumption, I construct a measure of total economic expenditure on gasoline and fuel oil using prices and quantities from the EIA. While these are not the only uses of petroleum in the economy, these two sources account for roughly 65% of total product supplied in terms of barrels. The lack of price availability for the remaining products in the EIA measure prevents quantifying the total dollar value, however in terms of expenditure these two components probably account for an even larger percentage since both of these products are more highly refined than many of the other petroleum products and thus command higher prices. Figure 4 shows the two level of expenditures from 1983 to 2010. Personal consumption expenditure of gasoline and fuel oil accounts for a relatively stable share of total economic consumption which varies from 60% to 70%. The fact that a very large portion of total gasoline and fuel oil consumption is accounted for by personal consumption suggests that considering oil as a consumption good rather than an input to production is not an unreasonable approach.
While the consumption based asset pricing literature traditionally relies on nondurables and services as the measure of consumption, recent work by Yogo (2006) and Yang (2010) emphasizes the importance of durable consumption for explaining asset prices. Yang in particular finds that in a long run risk setting, the high persistence of Durable consumption can explain much of the observed equity premium. I follow Yang (2010) and consider consumption as an equally weighted Cobb-Douglas aggregate of the stock of durable goods and expenditure on nondurables and services (excluding energy consumption). I find that this measure does a better job of explaining oil prices than nondurable consumption, and that the added persistence of consumption growth is important in explaining observed features of the futures curve. Following Yogo (2006) I construct a quarterly series for the stock of durable consumption using yearly data for the stock of consumer durables and
quarterly data for expenditure on durable goods.

Data for oil prices is historical data for futures contracts of horizons out to twelve months in Crude Light Sweet oil traded on the NYMEX, and the real spot price of oil is the West Texas Index deflated by a measure of the price of the aggregate consumption good. This price measure is constructed using price levels from the NIPA survey. Table I reports summary statistics and correlations for the growth rates of the pertinent data. Futures price data is available from 1987 to 2010. When considering dynamics of the futures curve I focus on the period prior to the financial crisis to avoid problems of outliers due to very large moves in oil prices over these periods. Including them does not change any of the qualitative results. Formal tests for a change in expected persistence (described in Section A) strongly support a structural break in the data in October of 2003. I therefore consider two samples, from 1987 to October 2003, and from November 2003 to June of 2008, prior to the financial crisis.

Table I: Growth Rate Summary Statistics

Summary statistics for quarterly growth rates of relevant variables. Cobb-Douglas aggregate is an equally weighted aggregate of the stock of durable goods and the sum of nondurables and services. Nondurable consumption excludes energy goods. The real spot price of oil is calculated as the WTI deflated by the BEA price levels for consumption.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Mean (%)</th>
<th>SD (%)</th>
<th>Autocorrelation</th>
<th>Spot Price</th>
<th>Nondurables</th>
<th>Durables</th>
<th>Aggregate</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real Spot Price of Oil</td>
<td>0.98</td>
<td>16.80</td>
<td>0.11</td>
<td>0.09</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nondurables and Services</td>
<td>0.44</td>
<td>0.45</td>
<td>0.54</td>
<td>0.13</td>
<td>0.43</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stock of Durable Goods</td>
<td>0.99</td>
<td>0.48</td>
<td>0.93</td>
<td>0.13</td>
<td>0.83</td>
<td>0.86</td>
<td></td>
</tr>
<tr>
<td>Cobb-Douglas Aggregate</td>
<td>0.72</td>
<td>0.40</td>
<td>0.81</td>
<td>-0.10</td>
<td>0.46</td>
<td>0.22</td>
<td>0.24</td>
</tr>
<tr>
<td>Personal Oil Consumption</td>
<td>0.29</td>
<td>0.18</td>
<td>0.01</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

B Intratemporal Utility and the Oil Price

The model as written implies two cointegrating relations. The first, which I will refer to as the Intratemporal relation, arises from the functional form of $V_t$ and implies that a linear combination the two types of consumption, $\frac{1}{\rho}(c_t - \eta o_t)$, will be cointegrated with $p_t$. This simple version of the model implies that they are in fact equal, but to test this empirically I will test that difference between $p_t - \frac{1}{\rho}(c_t - \eta o_t)$ is a stationary process. I find strong
evidence that this is the case, and that not only is the difference a stationary process, but that the predicted spot price $\frac{1}{\rho}(c_t - \eta o_t)$ provides an excellent proxy for the real spot price of oil. This result is crucial for motivating the model, since the consumption dynamics can only have meaningful implications for oil prices if there exists a relation between levels of consumption and the spot price of oil. Documenting the existence and strength of this relation is one of the main empirical contributions of this paper, and provides a starting point for which to consider the relation between consumption and oil price risk.

Cointegration analysis is a common tool in the study of oil or gasoline prices. Several studies such as Bentzen and Engsted (1993) and Ramanathan (1999) seek to estimate both long run and short run elasticities of consumption to prices using methods similar to those I use here. Typically these analyses begin by proposing a demand function for oil where the log of economy-wide oil or gasoline consumption is assumed to be a linear function of the logs of other economic variables, most often personal income and the price of oil. Since I am interested in pricing assets in the consumption based long run risk framework, the relation I focus on involves personal consumption, $o_t$, and is implied by the first order condition of an optimizing representative agent with utility over two goods.

I follow Yogo (2005) and estimate a cointegrating relation between the log of oil prices and measures of consumption and oil consumption. A simple method for doing this is the Dynamic OLS method described by Stock and Watson (1993), where equation (3) is estimated, including both leads and lags of the dependent variables, resulting in the following form for the regression.

$$p_t = \beta_0 + \beta_1 c_t + \beta_2 o_t + \sum_{t=-k}^{k} \Gamma_{1,k} \Delta c_{t+k} + \sum_{t=-k}^{k} \Gamma_{2,k} \Delta o_{t+k}$$ (25)

The coefficients are related to the parameters of the utility function $V_t$ by $\beta_1 = \frac{1}{\rho}$ and $\beta_2 = \frac{2}{\rho}$. This regression model is identical to that of Bentzen, with personal aggregate consumption and personal oil consumption standing in for personal income and economy wide oil consumption. It is worthwhile to note here the implications of considering oil directly as a consumption good. While clearly consumers do not consume crude oil, and ultimately
I will be concerned with pricing futures contracts for delivery of crude oil, there is a very tight relation between crude oil prices and the price of gasoline, which does directly enter the consumer’s consumption basket. More importantly, I find that data on personal consumption of oil products taken from Bureau of Economic Analysis’ NIPA tables, provide substantial improvement in explanatory power for oil prices over typical measures of crude oil and gasoline consumption taken from the Energy Information Association (EIA). Aggregate consumption performs equally as well as personal income in predicting oil prices.

When doing the regressions with consumption, I divide the consumption data by estimates of the U.S. population taken from census data. In order to account for changes in the efficiency of converting oil to consumption utility, I adjust the level of oil consumption by the multiplying it by average miles per gallon taken from the Bureau of Transportation Statistics. The assumption underlying this adjustment is that the consumption good is not actually gasoline, but rather miles driven. Therefore, I also adjust the price of oil by miles per gallon. Therefore, in the regression of Equation (25), I substitute $p_t$ with $(p_t - \log(mpg_t))$, and $o_t$ with $(o_t + \log(mpg_t))$. These adjustments do not add significant volatility to the series, but they do adjust the growth trends, which are important in determining the cointegrating relation. I do not perform these adjustments when using the data for economy wide oil consumption to be consistent with other studies, however performing this adjustment for this data does not significantly improve the estimates.

I estimate this regression using two different measures of aggregate consumption, both consumption of nondurable goods and services and a Cobb-Douglas aggregate of nondurable goods and services and the stock of durable goods constructed as in Yogo [2006]. I also include two different measures of the consumption of oil. The first, following Bentzen and others, is the economy-wide measure of product supplied from the EIA, the second is the measure of energy product consumption (including gasoline and heating oil) from NIPA consumption data. For comparison I also estimate the regression using personal income and GDP in place of consumption. Table II reports these regressions for 1987 to 2010, the period for which I have futures data, as well as regressions of the oil price on levels and leads and lags of each variable individually.
Table II: Oil Prices and Economic Variables

Estimation of Stock and Watson (1993) regressions of log real spot price on logs of economic variables. The spot price is the WTI index adjusted by the price of the aggregate consumption good. \( mpg_t \) is the miles per gallon of the U.S. passenger car fleet. GDP is log of U.S. real GDP, \( c_t \) is the log of the aggregation of durables and nondurables. Income is log of personal income taken from the NIPA tables. \( o_{Personal}^t \) is the log of personal oil consumption of energy goods taken from the NIPA tables adjusted for U.S. passenger car fleet miles per gallon, \( o_{Total}^t \) is the measure of oil "Product Supplied" taken from EIA data. Personal oil consumption, household aggregate consumption and personal income are measured per capita. Regressions are performed with contemporaneous, as well two leads and lags of, differences. Coefficients on difference terms are suppressed. Standard errors are Newey-West with two lags.

(a) 1987-2010

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1) ((p_t - \log(mpg_t)))</th>
<th>(2) ((p_t - \log(mpg_t)))</th>
<th>(3) ((p_t - \log(mpg_t)))</th>
<th>(4) (p_t)</th>
<th>(5) (p_t)</th>
<th>(6) (p_t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_t )</td>
<td>3.099*** (0.247)</td>
<td>4.168*** (0.778)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log Income</td>
<td>3.556*** (0.403)</td>
<td></td>
<td>1.398 (1.073)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Log GDP</td>
<td></td>
<td>3.050*** (0.388)</td>
<td>-0.0702 (0.887)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( o_{Personal}^t )</td>
<td>-5.534*** (0.658)</td>
<td>-6.404*** (0.875)</td>
<td>-6.070*** (0.964)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( o_{Total}^t )</td>
<td></td>
<td></td>
<td>-7.381*** (2.026)</td>
<td>-0.309 (2.331)</td>
<td>2.759 (2.178)</td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.839 0.749 0.768</td>
<td>0.740 0.595 0.643</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

(b) 1972-2010

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1) ((p_t - \log(mpg_t)))</th>
<th>(2) ((p_t - \log(mpg_t)))</th>
<th>(3) ((p_t - \log(mpg_t)))</th>
</tr>
</thead>
<tbody>
<tr>
<td>( c_t )</td>
<td>3.934*** (0.251)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Income</td>
<td></td>
<td>4.247*** (0.300)</td>
<td></td>
</tr>
<tr>
<td>GDP</td>
<td></td>
<td></td>
<td>4.066*** (0.319)</td>
</tr>
<tr>
<td>( o_t )</td>
<td>-7.442*** (0.459)</td>
<td>-7.194*** (0.480)</td>
<td>-7.961*** (0.594)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.725 0.692 0.653</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1
The two things to note in this table are that the measurement of oil consumption from NIPA data does a much better job of explaining oil prices than the measure of consumption obtained from the EIA. Secondly is that, in terms of $R^2$, the consumption aggregate of Durable and Nondurable goods explain more of the variation in prices than the two standard measures in the literature, GDP and Personal Income. Augmented Dickey-Fuller tests (not reported) of the residuals of these regressions strongly reject the presence of a Unit Root in the case of personal consumption of oil, indicating a cointegrating relation between oil prices and the economic variables. In order to illustrate the goodness of fit of this model Figure 5 graphs the predicted values from a simple regression of the log of the oil prices on the logs of aggregate consumption and energy consumption from 1965 to 2010. The estimates of these simple regressions on the longer sample are not statistically different from the estimates from the Stock and Watson regressions on the shorter sample. This strong evidence of a stable relation between oil prices and consumption suggests that it is reasonable to start with a model of consumption dynamics when considering the behavior of oil prices.
Predicted prices are the predicted value from the regression \( p_t - \log(mpg_t) = \beta_0 + \beta_1 c_t + \beta_2 (o_t + \log(mpg_t)) + \epsilon_t \), where \( p_t \) is the log of the WTI spot price adjusted by CPI excluding energy costs, \( c_t \) is a CES aggregation of the stock of durable consumption and expenditure nondurable consumption (excluding energy goods), and \( o_t \) is the measure of energy good consumption from the NIPA survey. Consumption measures are adjusted by the U.S. Population. \( mpg_t \) is the average miles per gallon of the U.S. passenger car fleet taken from the Bureau of Transportation Statistics.

C Oil Price Shocks and Future Consumption Growth

The other important relation between consumption and oil prices from the LRR perspective is the power of oil prices in predicting future consumption growth. Hamilton [2008] shows that regressing GDP growth on lagged innovations to oil prices from 1972 - 2005 indicates that positive oil price increases negatively predict future GDP growth. I perform identical regressions using my measure of aggregate consumption in place for 1972 - 2010 and confirm this result. Results are reported in Table III. Therefore, the result that consumption growth predicts negative aggregate consumption is not unique to my choice of sample period.
Table III: Oil Price Shocks and Future Consumption Growth: 1972 - 2010

Regression of growth of Cobb-Douglas aggregate of durable and nondurable consumption on its own lagged growth and lagged oil price innovations. \( p_t \) is the log of the real spot price, as measured by the WTI spot price deflated by the price of consumption goods. Data is quarterly frequency. Standard Errors are Newey-West with four lags. P-value is reported for F-test of the null hypothesis that the values of the \( \Delta p_{t-i} \) terms are equal to zero.

| VARIABLES | \( \Delta c_t \) | \( \Delta c_{t-1} \) | 0.648*** 
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(0.098)</td>
<td></td>
</tr>
<tr>
<td>( \Delta c_{t-2} )</td>
<td>0.077</td>
<td>(0.107)</td>
<td></td>
</tr>
<tr>
<td>( \Delta c_{t-3} )</td>
<td>0.167</td>
<td>(0.106)</td>
<td></td>
</tr>
<tr>
<td>( \Delta p_{t-1} )</td>
<td>-0.002</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>( \Delta p_{t-2} )</td>
<td>-0.002*</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>( \Delta p_{t-3} )</td>
<td>-0.001</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>( \Delta p_{t-4} )</td>
<td>-0.003***</td>
<td>(0.001)</td>
<td></td>
</tr>
<tr>
<td>Constant</td>
<td>0.000795</td>
<td>(0.0005)</td>
<td></td>
</tr>
<tr>
<td>R-squared</td>
<td>0.689</td>
<td></td>
<td></td>
</tr>
<tr>
<td>P-Value Oil Price F-Test</td>
<td>0.0015</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

III Changes in Oil Prices

Expected log oil prices can be expressed in the model as

\[
E_t[p_{t+1}] = (1 + \pi_c - \eta \pi_o) p_t + (1 - \Phi_x \eta) x_t
\]  

Therefore changes in spot and futures price dynamics should reflect changes in the underlying consumption process. For instance, a decrease in \( \pi_o \) would lead to a larger value for the AR(1) coefficient for spot prices, \( 1 + \pi_c - \eta \pi_o \), and hence less mean reversion in oil prices. A change in \( \Phi_x \) leads to differences in how expected spot prices respond to changes in \( x_t \). I show that over the period from 2004 to 2008, the dynamics of oil prices and oil futures prices were very different than in prior years, and that these changes are
consistent with unresponsive oil supply, represented in the model by a small values of \( \pi_o \) and \( \Phi_x \).

The time period I am focusing on for this analysis is the 21 year period from 1987 to just prior to the financial crisis. Over this period I have data on futures prices out to 12-month horizons. As Figure 5 shows, following the Oil Price Crash of 1986, there is roughly 12 year period of remarkable stability in oil prices. Around 2003, prices began to rise, increasing by 400% over the next 5 years, before falling sharply following the financial crisis of 2008. Therefore, though the start of this period is dictated by the availability of data, even in the absence of this constraint this time period is a potentially interesting one. I do not include the period after the crash for two reasons. The first is that extremely high volatility tends to produce outliers in the data which dominate in a short sample, and the second it appears that the drop in demand has freed up large amounts of excess oil capacity. However, most of the results shown here are robust to including the two years since the crisis.

A Changes in the Persistence of Oil Prices

The main empirical observation which motivates splitting the sample into different subperiods is a change in the mean-reversion of oil prices. The existence of mean reversion in oil prices is a topic which has received substantial attention in the macroeconomic literature. Some studies, such as Routledge, Seppi, and Spatt (2000) and Schwartz (1997), find evidence of mean reversion in oil using high frequency data in the 1990s. However, Hamilton (2008) describes oil prices over the period of 1973 to 2008 as a pure random walk based on the results of an Augmented Dickey-Fuller test using quarterly data. More recently, Dvir and Rogoff (2009) employ the test of Harvey, Leybourne, and Taylor (2006) to detect structural changes in the price of oil from an I(0) to an I(1) process and vice versa. They examine a much longer horizon, and test for a single change in behavior from 1881 to 2008 and find evidence of a change of oil from an I(0) to an I(1) process in 1973.

There are two ways which I test for changes in persistence. The first is to employ a regression technique similar to that of Bessembinder, Coughenour, Seguin, and Smoller
(1995), where changes in long term futures prices are regressed on the innovations in the spot price. High mean reversion should imply that longer term contracts move less in response to a change in the level of prices. The second is to employ standard time series tests for a unit root in the spot price.

From the perspective of an asset pricing model, such as the one presented in this paper, it is the agent’s expectation of the persistence of oil that is the important determinant for prices of risk. Therefore, the existence of futures prices, which reflect expectations of future prices, is a powerful tool for observing changes in expected persistence.

I follow convention and define the excess return on a futures contract with \( j \) months to maturity as:

\[
r_{t+1}^j = f_{t+1}^j - f_t^j + \Delta p_t + \epsilon_{t+1}
\]  

(27)

Given my data for futures out to twelve months, I have observations for returns of futures with horizons from two months to twelve months. I ignore the return on the nearest term futures price to avoid issues of high volatility as the contract gets close to delivery. I perform a simple regression of the futures return at each maturity on the contemporaneous change in spot price.

\[
r_{t+1}^j = \gamma_0 + \gamma_1 \Delta p_t + \epsilon_{t+1}
\]  

(28)

To formally test for a change in the persistence of oil prices I use the test of Bai and Perron (1998) to test for a structural break in this regression, and find strong evidence for a structural break towards the end of 2003.\(^{10}\) For robustness, I also use the test of Busetti and Taylor (2004) on the time-series of spot prices and find evidence for a switch from I(0) to I(1) at the beginning of 2000. Since the relevant variable for the model is the expected persistence of oil prices, I split the sample at the later date. Note that all the

\(^{10}\)This break exists irregardless of whether the sample is cut off prior to the financial crisis of 2008 or extended to include more recent data. For the reasons mentioned in the introduction I focus on the period prior to the financial crisis.
Results are qualitatively similar for any breakpoint between 2000 and 2003. The later date also coincides with beginning of the time period characterized by low excess production capacity.

Results of the regressions for two month, six month, and twelve month futures in the two periods are reported in Table IV. These regressions confirm that the realized differences in persistence also lead to changes in expected persistence, with a coefficient of the longest term contract changing from 0.44 in the first period to 0.75 in the second.

Table IV: Regressions of Returns on Changes in Spot Price

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\gamma_0$</td>
<td>$\gamma_1$</td>
</tr>
<tr>
<td>$r^{2}_t$</td>
<td>0.006</td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>$r^{6}_t$</td>
<td>0.005</td>
<td>0.62</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>$r^{12}_t$</td>
<td>0.003</td>
<td>0.44</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.03)</td>
</tr>
</tbody>
</table>

Another way to test for persistence is to examine the time series behavior of spot prices. Table V shows standard unit root tests for the log of the real spot price. For the first sample both Augmented Dickey Fuller Tests and Phillips-Perron tests reject the null that oil prices contain a unit root, while being unable to reject for the second period. The table also reports the first order autocorrelation of the log of the spot price, estimated from an AR(1) regression. The autocorrelation is significantly higher in the second period.
Table V: Unit Root Tests of Oil Spot Prices

Phillips-Perron tests and Augmented Dickey Fuller tests for a unit root in \( p_t \), the log of the WTI spot using monthly data. \( P \)-values are in parentheses. \( A \) (∗) or (†) denote rejection of a unit root at the 5% and 10% significance levels.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Augmented Dickey-Fuller</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Z(t) )</td>
<td>-2.64∗†</td>
<td>-0.556</td>
<td>-0.88</td>
</tr>
<tr>
<td>( \text{P value} )</td>
<td>0.08</td>
<td>0.88</td>
<td>0.79</td>
</tr>
<tr>
<td>Phillips-Perron</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Z(\rho) )</td>
<td>-16.052†</td>
<td>-0.05</td>
<td>-3.29</td>
</tr>
<tr>
<td>( \text{P value} )</td>
<td>0.06</td>
<td>0.95</td>
<td>0.73</td>
</tr>
<tr>
<td>AC(1) ( p_t )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \text{Est} )</td>
<td>0.92</td>
<td>0.99</td>
<td>0.99</td>
</tr>
<tr>
<td>( \text{Std Err} )</td>
<td>0.03</td>
<td>0.03</td>
<td>0.01</td>
</tr>
</tbody>
</table>

B Changes in the Term Structure of Oil Futures

While the run up in prices over the second half of the sample was well publicized, what has not been as closely studied is the difference in the term structure of futures over these two periods. Panel A of Figure 1 graphs the average term structure of futures for each subperiod. What is noteworthy here is the switch of the curve from backwardation to contango over short horizons, and the change in the curvature of from concave to convex. While this second change may seem of little significance, it has important implications for expected returns and hence risk premia. The difference in futures prices for contracts at adjacent months can be expressed as

\[
f_t^{t+j} - f_t^{t+j-1} = -E_t[r_{t+1}^j] + E_t[f_{t+1}^{t+j-1} - f_t^{t+j-1}] \tag{29}
\]

This difference is decomposed into two pieces, the expected return, and the expected change in the futures price for a contract maturing at date \( t + j - 1 \). Therefore, one possible candidate for explaining changes in the term structure of prices is changes in expected returns. There are two important changes across the two periods. First, as Panel A in Figure 1 shows, the term structure of prices is more upward sloping, or exhibits ”contango” in the second period, as opposed to the downward sloping ”backwardation” of the first period. In order to test the significance of this changes, the difference between
the 2-month and 1-month futures prices \((f_{2,t} - f_{1,t})\) is regressed against current and lagged futures returns to attempt to capture variation coming from changes due to expected mean reversion in spot prices. The constant term then provides a measure of the average expected return. Table VI reports these regressions, and yields a significantly positive constant in the regression in the second period compared to the significantly negative constant in the first.

**Table VI: Futures Slope Regressions**

Regressions of the log difference of two and one month futures prices on current and lagged futures returns. Standard errors are Newey-West with 6 lags.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(f_{2,t} - f_{1,t})</td>
<td>(f_{2,t} - f_{1,t})</td>
</tr>
<tr>
<td>(r_{1,t})</td>
<td>-0.08***</td>
<td>-0.07***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>(r_{1,t-1})</td>
<td>-0.07***</td>
<td>-0.05***</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>(r_{1,t-2})</td>
<td>-0.04*</td>
<td>-0.07***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>(r_{1,t-3})</td>
<td>-0.03*</td>
<td>-0.05***</td>
</tr>
<tr>
<td></td>
<td>(0.02)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>(r_{1,t-4})</td>
<td>-0.05***</td>
<td>-0.03**</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>(r_{1,t-5})</td>
<td>-0.05***</td>
<td>-0.03**</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>(r_{1,t-6})</td>
<td>-0.04***</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(0.01)</td>
<td>(0.01)</td>
</tr>
<tr>
<td>Constant</td>
<td>-0.0034**</td>
<td>0.011***</td>
</tr>
<tr>
<td></td>
<td>(0.0017)</td>
<td>(0.003)</td>
</tr>
</tbody>
</table>

R-squared | 0.485 | 0.504 |

Standard errors in parentheses

*** p<0.01, ** p<0.05, * p<0.1

The second change involves the term structure of realized returns. Panel B in Figure 1 graphs average log returns over the two subperiods. Returns are increasing in maturity in the second period, as opposed to decreasing in the first. Holding the second term of Equation (29) constant, the decreasing expected returns of the first period imply a convex term structure of future prices, while the increasing expected returns of the second period imply a concave term structure, which is precisely what we see.

While the returns to commodity futures are highly volatile like any asset, they are
also highly correlated with futures prices at other maturities. Therefore, when examining
differences between levels of expected return across the term structure, the relative returns
are considerably less volatile than the absolute returns, and inference can be made at much
shorter time horizons than would normally be required when considering the return on
a single asset. This is especially important in this setting, as I am interested in making
statements about changes in risk premia using merely 6 years of data.

This feature of futures prices, that the added dimension of returns across the term
structure gives extra power in identifying changes in the pattern of expected returns, has
been mostly overlooked in the literature. Many studies, such as Fama and French (1987)
and Gorton, Hayashi, and Rouwenhorst (2007b), examine the futures basis, or the "slope"
of the futures term structure, as a possible predictor of either changes in spot price or
returns on the nearest futures contract. While these are obviously related issues to this
analysis, they are focused on explaining the return to the contract of a single maturity,
rather than studying the term structure of expected returns.

In order to assign statistical significance to the observed differences of returns across I
estimate the following simple regression of expected returns on the maturity of the futures
contract.

\[ E[r_j^t] = \beta_0 + \beta_j j + \epsilon_j \]  \hspace{1cm} (30)

I estimate the coefficients using the Fama and MacBeth (1973) procedure. While this
procedure capitalizes on the comovement in returns by essentially allowing for a time fixed
effect, it does not account for the fact that when prices are rising, the short end of the
futures curve tends to increase more than the longer term contracts as evidenced by Table
IV. This effect creates larger standard errors in this setting. In order to control for it, I
define the following return

\[ \tilde{r}_j^t = r_j^t - \tilde{\gamma}_j r_t^{12} \]  \hspace{1cm} (31)
This is the return to a strategy of going long on a short term contract \( j \), and short a proportional position in the 12 month contract. The proportion, \( \tilde{\gamma}_j \) is determined by a 3 year rolling regression of \( r^j_t \) on \( r^{12}_t \). I then repeat the regression of equation 30. Results for the two regressions are reported in Table VII. For the basic regression the positive slope in the second period is significant with a p-value of 6%. For the regression using \( \tilde{r}^j_t \), this positive slope is highly significant with a p-value of 1%. The negative slope in the first period is not statistically significant at any conventional level for either regression.

### Table VII: Fama-MacBeth Regressions of Futures Returns on Time to Maturity

Regressions of expected return on maturity. \( r^j_t \) is the return on future of maturity \( j \). \( \tilde{r}^j_t \) is the return on maturity \( j \) controlling for the return on the 12 month maturity. Data is monthly. Errors are computed using the Fama-Macbeth procedure. † and ∗ denote significance at the 10% and 5% levels respectively.

<table>
<thead>
<tr>
<th>Dep Var</th>
<th>Constant</th>
<th>Maturity</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>1987 - 2003</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r^j_t )</td>
<td>0.105</td>
<td>-0.04</td>
</tr>
<tr>
<td>( (7.82) )</td>
<td>( (0.04) )</td>
<td></td>
</tr>
<tr>
<td>( \tilde{r}^j_t )</td>
<td>0.05</td>
<td>-0.020</td>
</tr>
<tr>
<td>( (0.34) )</td>
<td>( (0.03) )</td>
<td></td>
</tr>
<tr>
<td><strong>2004 - 2008</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( r^j_t )</td>
<td>2.5∗</td>
<td>0.090†</td>
</tr>
<tr>
<td>( (1.06) )</td>
<td>( (0.048) )</td>
<td></td>
</tr>
<tr>
<td>( \tilde{r}^j_t )</td>
<td>-1.27∗</td>
<td>0.138∗</td>
</tr>
<tr>
<td>( (0.41) )</td>
<td>( (0.04) )</td>
<td></td>
</tr>
</tbody>
</table>

### C Aggregate Volatility and the Slope of the Futures Curve

Across these periods it is interesting that there is a stable relation between aggregate volatility in the economy and the slope of the futures curve. In both periods high aggregate volatility corresponds to a more downward sloping term structure of oil futures. This is entirely consistent with the model, since in both periods high volatility of the aggregate consumption process creates a higher positive risk premium associated with oil prices, and
therefore a downward sloping futures curve.\textsuperscript{11}

Here another potential benefit of having the futures curve to measure changes in risk premia associated with changes in volatility is that the comparison of futures prices of different maturities controls for changes in the level of prices that may accompany shocks to volatility. For example, the tendency for equity prices to decline when option implied volatility increases might be explained by either a positive shock to volatility causing an increase in the required rate of return on equities, or by a negative shock to expected future cash flows that also results in an increase in volatility, possibly due to a leverage effect.\textsuperscript{12}

In the model, the relevant state variables is $\sigma_t$, which to directly observe, since consumption growth available at only quarterly frequency, therefore following Drechsler and Yaron (2009) I use the lag of the CBOE VIX index and one lag of $\sigma_{S&P,t}$ to calculate expected market return volatility. I then perform the following regression in each half the sample to examine the relation of expected volatility and the slope of the term structure, which is defined as the difference between the log of the 12-month future and the 1-month future.

\begin{equation}
 f_{t}^{12} - f_{t}^{1} = \beta_{0} + \beta_{\sigma,S&P}E_{t}[\sigma_{S&P,t+1}] + \sum_{i=0}^{L} \beta_{\Delta p,t-i}^{i} \Delta p_{t-i}
\end{equation}

Lagged returns are again included to capture variation in the slope caused by movements in spot prices and the mean reverting nature of oil. Results are reported in Panel A of Table VIII. Not surprisingly, given the decrease in mean reversion, the lagged price movements have less effect on the slope of the futures curve in the second period. In the both periods, high volatility implies a more downward sloping term structure.

Kogan, Livdan, and Yaron (2009) also consider the conditional relation of spot price volatility to the absolute value of the slope of the futures curve. They find that, over

\textsuperscript{11}If stochastic volatility of the oil consumption good is included, an increase in this volatility will generate a more upward sloping curve when oil shocks are important in the second period. This relation in the data is inconclusive for 2004 - 2008. Extending the second period 2010 I do find evidence of this relation, however it is driven almost exclusively by the post crises period in which extremely high oil price volatility occurred during a period of extreme contango.

\textsuperscript{12}Eraker (2008) provides evidence that suggests the observed negative correlation between equity prices and implied volatilities can be explained by changes in required rates of return.
Regressions of the slope of the futures curve on the risk-free rate and the expected volatility of equity prices. Expected volatility of equity prices is calculated following Drechsler and Yaron (2009) using a regression of realized daily volatility of the returns of the S&P 500 on the lag of realized volatility and the CBOE VIX index. The slope is the log difference between the twelve month futures price and the 1-month futures price. Standard Errors are Newey-West with 6 lags.

Table VIII: Aggregate Volatility and the Futures Curve

<table>
<thead>
<tr>
<th>Period</th>
<th>Dep. Var.</th>
<th>$E_t[\sigma_{S&amp;P,t+1}]$</th>
<th>$r_{1,t}$</th>
<th>$r_{1,t-1}$</th>
<th>$r_{1,t-2}$</th>
<th>Constant</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1987-2003</td>
<td>$f_t^{12} - f_t^1$</td>
<td>0.45 (0.45)</td>
<td>-1.03 (0.07)</td>
<td>-0.35 (0.07)</td>
<td>-0.35 (0.07)</td>
<td>-0.25 (0.07)</td>
<td>0.01 (0.03)</td>
</tr>
<tr>
<td>2004-2008</td>
<td>$f_t^{12} - f_t^1$</td>
<td>0.60 (0.08)</td>
<td>-2.85 (0.08)</td>
<td>-0.26 (0.08)</td>
<td>-0.19 (0.08)</td>
<td>-0.16 (0.08)</td>
<td>-0.13 (0.06)</td>
</tr>
</tbody>
</table>

In the period 1985 - 2001, when the futures curve has either a large positive slope or a large negative slope, it predicts high volatility of spot prices. They explain this effect with a production model with constraints on the adjustment of supply in each period. When the producer is adjusting supply to respond to a price shock, the adjustment constraint is binding and they are unable to respond to further changes in prices. Though this mechanism is not present in my model, it is worth noting that if production is no longer able to respond to prices at all, there will be no changing elasticity of supply and this effect will disappear. I confirm their result in the first half of the sample (regressions not reported), but find it is no longer present in the second subperiod. This is both consistent with their explanation of this result, and evidence for a lack of production response as a potential explanation for the changes in consumption dynamics that I focus on here.

IV Model Calibration

A Extensions to Intratemporal Utility

Before taking the model to the data, I augment the basic GCES intratemporal utility with an external habit to better match observed behavior of oil prices.
I note that the price of oil tends to be above the model predicted price when prices are rising, and vice versa. I define $\hat{\xi}_t$ to be the difference between the observed price of oil $p_t$ and the price implied by the agents' F.O.C., $\frac{1}{\rho}(c_t - \eta o_t)$, where $\rho$ and $\eta$ and taken from the original Stock and Watson regression. I perform the following regression:

$$\hat{\xi}_t = \alpha \xi + \sum_{i=0}^{n} \beta \xi_i \Delta (c_{t-i} - \eta o_{t-i}) + \epsilon_{\xi,t}$$  \hspace{1cm} (33)$$

The results of this regression are shown in Table IX. Adding changes in the relative level of consumption provides significant extra explanatory power to explain prices.

Economically this empirical result implies a short run elasticity of demand for oil consumption that is significantly lower than the long term elasticity. In an endowment setting this can be accommodated by incorporating a habit in oil consumption. This behavior in prices can be created in the model when $\xi_t$ is the inverse of a relative external habit for oil consumption, similar to Ravn, Schmitt-Grohe, and Uribe (2005), so that utility is given by.

$$V_t(C_t, O_t) = \left[(1 - a)C_t^{1 - \frac{1}{\rho}} + a \left(\frac{O_t}{X_t}\right)^{1 - \frac{2}{\rho}}\right]^{\frac{\rho}{\rho - 1}}$$ \hspace{1cm} (34)$$

Where

$$X_t = e^{-\xi_t}$$ \hspace{1cm} (35)$$

With this specification the optimizing behavior of the agent implies

$$p_t = const + \frac{1}{\rho}(c_t - \eta o_t) - \left(1 - \frac{\eta}{\rho}\right) \xi_t$$ \hspace{1cm} (36)$$
Table IX: Observed Oil Price and Innovations to Implied Oil Price

$\xi_t$ is the observed difference between the spot price of oil and the observed value of $\frac{1}{\rho}(c_t - \eta o_t)$. Results are reported for a regression of $\xi_t$ on lags of innovations to the value of $c_t - \eta o_t$. Values of $\rho$ and $\eta$ are those calculated in regression (1) of Table IIa. Data is quarterly frequency from 1987 - 2010. Standard Errors are Newey-West with three lags.

<table>
<thead>
<tr>
<th>VARIABLES</th>
<th>(1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta(c_t - \eta o_t)$</td>
<td>0.859** (0.383)</td>
</tr>
<tr>
<td>$\Delta(c_{t-1} - \eta o_{t-1})$</td>
<td>0.876** (0.407)</td>
</tr>
<tr>
<td>$\Delta(c_{t-2} - \eta o_{t-2})$</td>
<td>0.912** (0.377)</td>
</tr>
<tr>
<td>$\Delta(c_{t-3} - \eta o_{t-3})$</td>
<td>0.776* (0.464)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.172</td>
</tr>
</tbody>
</table>

Though habits are usually specified to reflect the innovations of a single consumption good, given the high complementarity of oil to aggregate consumption, it is reasonable to assume that innovations to aggregate consumption also will affect the level of habit. Or equivalently that the habit is in effect a habit for the relative level of oil consumption to aggregate consumption rather than the absolute level of oil consumption. Therefore to be consistent with the empirical data, I specify the following dynamics for the level of habit

$$\xi_{t+1} = \rho_\xi \xi_t + \frac{1 - \epsilon}{\epsilon} \Delta(c_{t+1} - \eta o_{t+1})$$ (37)

This specification of price has little qualitative effect on patterns of returns in the model. To show how this extension affects the exposure of oil prices to the underlying shocks, Figure 6 shows impulse response functions with the extended formulation of prices. The patterns are qualitatively the same as in Figure 3, but the magnitudes of exposure are larger. Empirically, adding this allows for a better fit of prices as evidenced by the regression in Table IX, and also allows the model to better match magnitudes of observed expected returns.
Figure 6: Model Impulse Response Functions: With External Habit

Impulse response function of logs of aggregate consumption ($c_t$), oil consumption ($o_t$), and the oil price ($p_t = \frac{1}{\rho}(c_t - \eta o_t) + \xi_t$) to innovations to oil consumption and the expected growth of aggregate consumption.

**Period 1:** $\pi_o = .1$ and $\Phi_x = \frac{1}{\eta}$

(a) Negative shock to $e^o_t$

(b) Positive shock to $e^o_t$

**Period 2:** $\pi_o \approx 0$ and $\Phi_x = 0$

(c) Negative shock to $e^o_t$

(d) Positive shock to $e^o_t$
B Calibrations

I solve the full model numerically using standard projection methods with the assumption that the value function is linear in the state variables\textsuperscript{13} and I calibrate two different scenarios for the model to match observed moments from the two subperiods of the sample (January 1987 to September 2003 and October 2003 to June 2008). Importantly, the only parameters are I allow to vary between the two scenarios are $\pi_o$ and $\Phi_x$, the two parameters which govern oil consumption response. The remaining parameters are constant across the two calibrations.

The long-run risk literature has generally relied on risk aversions between 5 and 25 and IES between 1.5 and 2.5. To be consistent with these values I set $\gamma$ equal to 18, and the IES $\psi$ equal to 2. I constrain $\Phi_x$ to be zero in the second period, while setting $\Phi_x = \frac{1}{\eta}$ in the first period so that long run consumption growth has no impact on the expectation of the long run oil price. I set $\frac{1}{\rho} = 3.5$ and $\eta = 1.75$ to match the values from the intratemporal cointegrating regression. Remaining parameters are chosen to match important model moments of oil prices, most notably the persistence and volatility of aggregate consumption and oil consumption, and the volatility of prices.

The parameters for the endowment process of $c_t$ and $x_t$ are generally in line with the LRR literature. One small departure from the normal convention is that shocks to $e_t^c$ and $e_t^x$ are allowed to be coordinated. This is done to generate a stronger positive risk premia for oil prices in the first period, but comes at the expense of reducing the dispersion in risk premia across the term structure in the second period. Volatility and persistence are of aggregate consumption are still close to those in the data. Table X provides parameters for the estimated models.

Figure 7 provides graphs of expected futures curves for the two different calibrations of the model each containing three panels. I am primarily concerned with matching the relative changes across the term structure. I do not view the aggregate return on on oil spot prices over such short periods as true indicators of risk. I therefore normalize the

\textsuperscript{13} This is analogous to the log-linear closed form approximations but removes need for separately approximating the GCES function
Table X: Model Parameters

<table>
<thead>
<tr>
<th>Description</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Utility</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Interemporal Elasticity of Substitution</td>
<td>$\psi$</td>
<td>2.5</td>
</tr>
<tr>
<td>Risk-Aversion</td>
<td>$\gamma$</td>
<td>18</td>
</tr>
<tr>
<td>Discount Factor</td>
<td>$\beta$</td>
<td>0.999</td>
</tr>
<tr>
<td>Elasticity of Oil / Aggregate Substitution</td>
<td>$\rho$</td>
<td>3.00</td>
</tr>
<tr>
<td>Necessity of Oil Good</td>
<td>$\eta$</td>
<td>1.75</td>
</tr>
<tr>
<td>External Habit Persistence</td>
<td>$\rho_\xi$</td>
<td>0.97</td>
</tr>
<tr>
<td>External Habit Elasticity</td>
<td>$\psi_\xi$</td>
<td>0.4</td>
</tr>
<tr>
<td><strong>Consumption</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Average Growth of Consumption</td>
<td>$\mu_c$</td>
<td>0.0025</td>
</tr>
<tr>
<td>Mean of Agg Consumption Volatility</td>
<td>$\sigma_{c,0}$</td>
<td>0.0011</td>
</tr>
<tr>
<td>Mean of Oil Consumption Volatility</td>
<td>$\sigma_o$</td>
<td>0.0052</td>
</tr>
<tr>
<td>Relative Volatility of Dividend Shocks</td>
<td>$\phi_d$</td>
<td>25</td>
</tr>
<tr>
<td>Relative Volatility of Expected Growth</td>
<td>$\phi_x$</td>
<td>0.10</td>
</tr>
<tr>
<td>Cointegrating Coefficient for Agg. Cons.</td>
<td>$\pi_c$</td>
<td>-0.0004</td>
</tr>
<tr>
<td>Persistence of $x_t$ Process</td>
<td>$\rho_x$</td>
<td>0.984</td>
</tr>
<tr>
<td>Correlation of Short and Long-Term Cons. Growth</td>
<td>$\text{corr}(c_t, c_{t+1})$</td>
<td>0.20</td>
</tr>
<tr>
<td>Loading of Dividend Growth on $x_t$</td>
<td>$\Phi_d$</td>
<td>2.25</td>
</tr>
<tr>
<td>Volatility of Volatility</td>
<td>$\sigma_w$</td>
<td>0.00000006</td>
</tr>
<tr>
<td>Persistence of Stochastic Volatility</td>
<td>$\nu$</td>
<td>0.95</td>
</tr>
<tr>
<td><strong>Oil Consumption Response</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cointegrating Coefficient for Oil Cons.</td>
<td>$\pi_o$</td>
<td>0.011</td>
</tr>
<tr>
<td>Reaction of Oil Consumption to Expected Growth</td>
<td>$\Phi_x$</td>
<td>$\frac{1}{1.75}$</td>
</tr>
</tbody>
</table>

return on the one month future to be equal to the observed return and then observe the relative pattern of returns at different term structures. Additionally, the model struggles on two dimensions, the first is that in both periods the model generates a term structure of
oil futures that is more upward sloping that that observed in the data. This is due to both
the log-normality of prices which generate an upward slope, and the non-homotheticity of
preferences. The value of $\eta > 1$ requires an upward drift in prices to create a stationary
value function. Additionally the model comes short of matching the magnitudes of the
observed slopes in the term structure of returns.

While this may be interpreted as a weakness of the model, it is important to understand
that the endowment process considered here is a simplification of a process generated by
the interaction of production, storage, and consumption of oil in the economy. Though
the futures curve is cut at a year due to data availability, the model returns keep rising
over the long term due to the highly persistent growth in $x_t$. Therefore the observed curve
could potentially be explained by a storage response occurring over the first year, where
oil prices rise as storage is increased in anticipation of higher future prices\(^{14}\).

The interpretation here then is that the qualitative effects with reasonable quantities can
be obtained in this framework, however this framework is not a replacement for reduced-
form models of the term structure, and there is still much that can be potentially learned
by extending this framework to a model of commodity production and storage.

Table XI gives sample moments and model moments of aggregate consumption dynam-
ics, and Table XIII gives the same for asset pricing dynamics. The model data for both
of these tables is generated by simulating 40 year samples, with the mean, and 5th and
95th percentiles, reported. Data moments are reported for 1970 - 2010. Table XII reports
sample and model moments for oil prices over the two sample periods. Model moments
for this table are generated with 15-year samples to illustrate the ability of the model to
generate differences in relative returns which are significant even over smaller sub-samples.

The results in Table XII show the model is able to generate an increase in the slope of
the term structure of prices, and switch in the term structure of returns from downward
sloping to upward sloping. In terms of significance, the upward sloping term structure of
returns is highly significant, consistent with the data. The raw difference in the slope of
prices is not significant, but (unreported) regressions analogous to those in Table VI yield

\(^{14}\)See Dvir and Rogoff (2009) for a description of this storage response to expectations of future price growth
Figure 7: The Term Structure of Crude Oil Futures: Model and Data

Observed average future prices, returns, and volatilities and the corresponding averages generated by the model. Lines represent actual data and solid shapes represent the model generated curves. For both futures and returns, the nearest maturity model moment is normalized to equal the observed moment in the data.

Panel A: Average Log Price

Panel B: Average Log Excess Return

Panel C: Return Volatility

Table XIII reports general asset pricing dynamics including the overall equity premium for the two models. Most interesting is the significant reduction in the equity premium driven by persistent oil prices acting as a significant counterweight to shocks to long-run

a significant increase in the average slope at a 10% level.
growth. The long-run nature of growth shocks in these models makes them particularly susceptible to oil prices which rise and fall with aggregate economic activity, and therefore the risk associated with them is substantially decreased in the second period. Further more, the raw difference in the numbers actually underestimates this effect, since the increased risk associated with shocks to oil consumption actually works to drive up this number.\textsuperscript{15}

The overall effect is not only a lowering of aggregate risk, but a shifting of risk away from other growth factors and into oil price risk, a result which could have important implications in the cross-section of equity returns. Though these implications are very difficult to test over the short sample period, this effect may become more apparent in the coming years if oil prices continue to behave in this manner.

Table XI: Data and Model Sample Moments: Aggregate Consumption

Calibrated model and data moments for consumption. \( c_t \) is a Cobb-Douglass aggregate of durable and nondurable consumption. Durable consumption growth is calculated as the growth in stock of durable goods from the NIPA consumption survey. Nondurable consumption is the sum of nondurables and services excluding energy goods.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1972 - 2010</td>
<td>Estimate</td>
<td>Std Error</td>
</tr>
<tr>
<td>Consumption Growth (Quarterly)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( E[\Delta c_t] )</td>
<td>0.72 (0.09)</td>
<td>0.74 -0.02 1.50</td>
<td>0.74 0.03 1.51</td>
</tr>
<tr>
<td>( \sigma(\Delta c_t) )</td>
<td>0.40 (0.02)</td>
<td>0.41 0.30 0.56</td>
<td>0.41 0.30 0.56</td>
</tr>
<tr>
<td>( AC(1)\Delta c_t )</td>
<td>0.81 0.04</td>
<td>0.77 0.61 0.88</td>
<td>0.77 0.61 0.89</td>
</tr>
</tbody>
</table>

Table XIV gives the result of the regressions in the model corresponding to the regressions of Table VIII. The regressions in the model have the same qualitative pattern as those in the data.

\textsuperscript{15}Oil consumption shocks generate roughly 2\% of the overall equity premium in Model 1, and 8\% in Model 2.
Table XII: Data and Model Sample Moments: Oil Prices and Oil Consumption

Calibrated model and data moments for oil consumption and prices. Data for oil consumption is from consumption of "Energy Goods" in the NIPA survey. The spot price of oil is the WTI spot price deflated by the CPI excluding energy goods. Future prices are NYMEX futures for Crude Light Sweet oil.

<table>
<thead>
<tr>
<th>A. Oil Consumption (Quarterly)</th>
<th>Model 1</th>
<th></th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>Mean</td>
<td>Mean</td>
<td></td>
</tr>
<tr>
<td>S.E.</td>
<td>S.E.</td>
<td>S.E.</td>
<td></td>
</tr>
<tr>
<td>5%</td>
<td>95%</td>
<td>5%</td>
<td>95%</td>
</tr>
<tr>
<td>E[\Delta o_t]</td>
<td>0.41 (0.15)</td>
<td>0.42 (0.18)</td>
<td></td>
</tr>
<tr>
<td>\sigma(\Delta o_t)</td>
<td>1.31 (0.10)</td>
<td>0.82 (0.13)</td>
<td></td>
</tr>
<tr>
<td>AC(1)\Delta o_t</td>
<td>-0.32 (0.12)</td>
<td>0.25 (0.23)</td>
<td></td>
</tr>
</tbody>
</table>

| C. Oil Futures Slope (Monthly) |  |  |
|-------------------------------|  |  |
| E[f_{2,t} - f_{1,t}]          | -0.67 (0.14) | 0.46 (0.22) |
| \sigma(f_{2,t} - f_{1,t})     | 1.90 (0.10) | 1.5 (0.09) |

| C. Oil Futures Returns (Monthly) |  |  |
|-------------------------------|  |  |
| E[r_{2,t}]                     | 0.91 (0.66) | 2.1 (1.12) |
| \sigma(r_{2,t})                | 9.20 (0.47) | 7.76 (0.45) |
| E[r_{12,t}]                    | 0.53 (0.38) | 3.1 (0.89) |
| \sigma(r_{12,t})               | 5.30 (0.27) | 6.2 (0.63) |
| E[r_{12,t} - r_{2,t}]          | -0.38 (0.38) | 1.00 (0.46) |
| \sigma(r_{12,t} - r_{1,t})    | 5.30 (0.27) | 3.16 (0.32) |

V Conclusion

This paper highlights the importance of oil prices as a factor in the dynamics of expected consumption growth, yielding a rich set of implications for asset prices. A change in the dynamics of the oil supply provides an explanation for observed changes in the term structure of oil futures prices. The changes in consumption dynamics which generate these changes also have much broader implications for risk in the overall economy. The decreased response of the oil supply to high oil prices lead to a highly persistent oil price, and in turn an increase in risk from oil price shocks. However the inability of the oil supply to respond to changes in consumption growth also means that high expected consumption growth generates high expectations of oil prices. In models of Long-Run Risk, shocks to growth are the primary force behind generating levels of risk sufficient to explain observed returns in asset prices. In the model presented here, an unresponsive oil price works to counter this risk and reduce the equity premium.
**Table XIII: Data and Model Sample Moments: Dividends and Returns**

Equity return, price, and dividend data are from CRSP. The risk free rate is the yield on the one-month Treasury Bill.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model 1</th>
<th>Model 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1972 - 2010</td>
<td>Estimate</td>
<td>Std Error</td>
</tr>
<tr>
<td>Dividend Growth (Annual)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[\Delta y_t]$</td>
<td>2.00</td>
<td>(2.24)</td>
<td>2.87</td>
</tr>
<tr>
<td>$\sigma(\Delta y_t)$</td>
<td>14.17</td>
<td>(1.58)</td>
<td>9.88</td>
</tr>
<tr>
<td>Price Dividend Ratio (Annual)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[P^m/Y]$</td>
<td>39.40</td>
<td>(1.03)</td>
<td>22.47</td>
</tr>
<tr>
<td>$\sigma(p^m - y)$</td>
<td>18.24</td>
<td>(1.19)</td>
<td>11.17</td>
</tr>
<tr>
<td>Returns (Annual)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$E[r_f]$</td>
<td>1.01</td>
<td>(0.03)</td>
<td>0.78</td>
</tr>
<tr>
<td>$\sigma(r_f)_t$</td>
<td>0.49</td>
<td>(0.02)</td>
<td>0.65</td>
</tr>
<tr>
<td>$E[r - r_f]$</td>
<td>5.80</td>
<td>(1.79)</td>
<td>6.60</td>
</tr>
<tr>
<td>$\sigma[r - r_f]$</td>
<td>15.32</td>
<td>(0.77)</td>
<td>12.42</td>
</tr>
</tbody>
</table>

**Table XIV: Slope Regressions in the Model**

Simulated regressions of the slope of the futures curve on aggregate volatility and past innovations in prices. $\sigma_{\text{market},t}$ is the conditional volatility of the stock market return for time $t + 1$ based on observed value of $\sigma_{c,t}$.

<table>
<thead>
<tr>
<th>Period</th>
<th>Dep. Var.</th>
<th>$\sigma_{\text{market},t+1}$</th>
<th>$\Delta p_t$</th>
<th>$\Delta p_{t-1}$</th>
<th>$\Delta p_{t-2}$</th>
<th>$\Delta p_{t-3}$</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model 1</td>
<td>$f_t^{12} - f_t^1$</td>
<td>-0.45</td>
<td>-0.34</td>
<td>-0.32</td>
<td>-0.31</td>
<td>-0.28</td>
<td>0.00</td>
</tr>
<tr>
<td>Model 2</td>
<td>$f_t^{12} - f_t^1$</td>
<td>-1.2</td>
<td>-0.12</td>
<td>-0.11</td>
<td>-0.11</td>
<td>-0.10</td>
<td>-0.01</td>
</tr>
</tbody>
</table>

These changes also have many implications outside of those considered here. For example, if companies’ stock returns have differential exposure to oil supply shocks or shocks to expected growth, the changes in prices of risks associated with these shocks will have implications in the cross-section of expected equity returns. With the ongoing concerns
about the oil supply in the coming decades, understanding how oil prices interact with models of macroeconomic risk is an important question, and is a promising area for future research.

References


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## VI Appendix

### Appendix A - Model and Solutions

In this section I derive approximate analytical solutions for the long run risk model with oil consumption. Lowercase variables represent logs.

**Intratemporal Utility**

Define
\[ C_t \equiv N_t^{1-\alpha} D_t^\alpha \] (38)

Where \( N_t \) is nondurable consumption expenditure excluding energy goods, and \( D_t \) is the stock of durable consumption goods. Define intratemporal utility as

\[ V_t(C_t, O_t) = \left[ (1 - a)C_t^{1-\frac{\theta}{\rho}} + aO_t^{1-\frac{\eta}{\rho}} \right]^\frac{\rho}{\rho - 1} \] (39)

Cobb-Douglas Approximation

In order to allow for analytical solutions to the model, I approximate the generalized CES utility function with a Cobb-Douglas utility function.

Let \( H_t = \frac{P_t O_t}{C_t} \) be the ratio of expenditure on oil to expenditure on the aggregate consumption good. \( \bar{H} \) is the sample average of \( H_t \).

Given the intratemporal first order condition. The generalized CES function can be rewritten as\(^{16}\):

\[ V_t = C_t \left( (1 + a)(1 + \frac{1 - \frac{1}{\rho}}{1 - \frac{\eta}{\rho}} H_t) \right)^{\frac{1}{1-\frac{\eta}{\rho}}} \] (40)

Taking a first order Taylor approximation of the log of intratemporal utility around the sample average ratio of expenditure gives

\[ v_t = c_t + \frac{1}{1 - \frac{1}{\rho}} \left( \log(1 - a) + (1 + \frac{1 - \frac{1}{\rho}}{1 - \frac{\eta}{\rho}} \bar{H}_t) + \frac{1 - \frac{1}{\rho}}{1 - \frac{\eta}{\rho}} \bar{H}_t \left( h_t - \bar{h}_t \right) \right) \] (41)

Since empirically the average value of \( H_t \) is roughly .025, the higher order terms are extremely small. Therefore I focus on the ability of the approximation in explaining the first order terms.

\(^{16}\)This holds for both Equations 1 and 34
The Cobb-Douglas approximation is

\[ \tilde{V}_t = C_t^{1-\tau} O_t^\tau \]  

(42)

where \( \tau = \frac{H}{1+H} \)

The approximation error to the first order terms is

\[ v_t - \tilde{v}_t = \text{constant} + (h_t - \bar{h}) \frac{\bar{H}^2}{1+H} \frac{(1-\eta)(\rho-1)}{(\rho-\eta)^2} \]  

(43)

Again, since \( \bar{H} \) is observed to be very small, this approximation error is negligible.

The marginal utilities of consumption and oil consumption under the generalized CES specification are

\[ V_{c,t} = \frac{V_t}{C_t} \frac{1}{1 + \left( \frac{1-\eta}{1-\rho} \right) H_t} \]  

(44)

\[ V_{o,t} = \frac{V_t}{O_t} \frac{H_t}{1 + \left( \frac{1-\eta}{1-\rho} \right) H_t} \]  

(45)

The marginal utilities under the approximation are

\[ \tilde{V}_{c,t} = \frac{\tilde{V}_t}{C_t} \frac{1}{1 + \bar{H}} \]  

(46)

\[ \tilde{V}_{o,t} = \frac{\tilde{V}_t}{O_t} \frac{\bar{H}}{1 + \bar{H}} \]  

(47)

Due to the small values of \( H_t \) observed in the sample, and the low variance of \( H_t \), this approximation performs well in terms of relative changes in marginal utility.

In fact given the small level of \( H_t \), \( C_t \) itself is a reasonable approximation of \( V_t \), since
most of the utility comes from the level of the basic consumption good. This approximation greatly simplifies the expressions so I will use this going forward.

**Intertemporal Utility**

I consider an agent with Epstein-Zin Preferences

Following Bansal and Yaron (2004) the log of the pricing kernel is

\[ m_{t+1} = \Theta \log \delta + \frac{\Theta}{\psi} \Delta c_{t+1} + (\Theta - 1) r_{g,t+1} \]

where \( r_{g,t+1} \) is the return on the aggregate consumption claim.

**Solving for the Return on the Consumption Claim**

For exposition I will assume that the shock terms are uncorrelated, though it is easy to allow for correlation between the shock terms.

I follow Bansal and Yaron (2004) and utilize the Campbell approximation for the return on the consumption claim.

\[ r_{g,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + \Delta c_{t+1} \]

Given these approximations, \( z_t \) is affine in three state variables, the predictable term \( x_t \) and the oil price error correction term \( p_t - \bar{p} \), and the stochastic volatility component, \( \sigma^2_{c,t} \)

\[ z_t = A_0 + A_1 x_t + A_2 (p_t - \bar{p}) + A_3 \sigma^2_{c,t} \quad (48) \]

To solve for the values of the \( A \) coefficients, I utilize the pricing equation.
\[ 1 = E_t[\exp(m_{t+1} + r_{g,t+1})] \] (49)

\[ 0 = E_t[m_{t+1} + r_{g,t+1}] + \frac{1}{2}\text{var}_t[m_{t+1} + r_{g,t+1}] \] (50)

The solutions for the coefficients are

\[ A_1 = \Phi_x + A_2\kappa_1(1 - \eta \Phi_x) - \frac{1}{2} \varphi_2 \sigma^2(1 + A_2(1 - \eta \Phi_x) \kappa_1)}{1 - \kappa_2(\rho_x - \sigma^2 \theta \kappa_2)} \] (51)

\[ A_2 = \pi_o \frac{\pi_o}{1 - \kappa_1(1 + \pi_o - \pi_o)} \] (52)

\[ A_3 = \frac{1/2 \Theta (1 + A_2 \kappa_1)^2 + A_2^2 \varphi_2}{1 - \kappa_2 \kappa_1} \] (53)

Innovations to the pricing kernel are

\[ m_{t+1} - E_t[m_{t+1}] = \left( -\theta \left( \frac{1}{\psi} - \tau (1 - \frac{1}{\psi}) + (\theta - 1) A_2 \kappa_1 \sigma_{c,t} \right) e_c^{t+1} \right) + (\gamma \theta + \theta \tau - \eta (\theta - 1)) A_2 \kappa_1 \sigma_{o,t} e_o^{t+1} + (\theta - 1) \kappa_1 A_1 \varphi_x \sigma_{c,t} e_{c,t}^{t+1} + (\theta - 1) \kappa_1 A_3 \sigma_{w,c} w_{c,t}^{t+1} \] (55)

Equivalently

\[ m_{t+1} - E_t[m_{t+1}] = -\lambda_c \sigma_{c,t} e_{c,t}^{t+1} - \lambda_o \sigma_{o,t} e_{o,t}^{t+1} - \lambda_x \varphi_x \sigma_{c,t} e_{c,t}^{t+1} - \lambda_{c,c} \sigma_{w,c} w_{c,t}^{t+1} \] (57)

Return on the consumption portfolio and the risk-free rate can then be solved for as in Bansal and Yaron [2004]
Oil Prices

Oil futures prices are linear in the state variables.

\[ f^j_t = \bar{p}_t + B^j_0 + B^j_x x_t + B^j_p (p_t - \rho \bar{p}) + B^j_{\sigma,c} \sigma_{t} \]  

(58)

The coefficients can be calculated by the following recursions.

\[ B^j_0 = B^{j-1}_0 + \nu_c \bar{\sigma}^2 \] 
\[ + \frac{1}{2} B^{j-1}_{\sigma,c} \bar{\sigma}^2_{w,c} + \lambda_{w,c} B^{j-1}_{\sigma,c} \sigma^2_{w,c} + B^{j-1}_p \lambda_{\sigma,o} \sigma^2_{o} \]  

(59)

\[ B^j_x = B^{j-1}_x \rho_x + B^{j-1}_p (1 - \Phi_x) \]  

\[ B^j_p = (1 + \pi_c - \pi_o) B^{j-1}_p \]  

\[ B^j_{\sigma,c} = \nu_c B^{j-1}_{\sigma,c} - (B^{j-1}_p) \lambda_{m,c} - (B^{j-1}_x) \phi_x \lambda_{m,x} \]  

(60)

Since \( f^0_t = p_t \). The initial values for the recursion are given by

\[ B^0_0 = \bar{p} \]  
\[ B^0_x = 0 \]  
\[ B^0_p = \frac{1}{\rho} \]  
\[ B^0_{\sigma,c} = 0 \]  

(62)

Equity Returns

The innovation to the market dividend \( y_t \) is represented by.
\[ \Delta y_{t+1} = \mu^y + \chi (x_t + \pi_c (p_t - \bar{p}_t)) + \varphi^y e_{t+1}^y \] (63)

The return on the market portfolio, \( r_{t+1} \), solves

\[ E_t [\exp (m_{t+1} + r_{t+1})] = 1 \] (64)

Again exploiting the Campbell approximation, \( r_{t+1} = \kappa_0^y + \kappa_1^y z_{t+1}^y - z_t^y + \Delta y_{t+1} \) and assume a linear form

\[ z_t = A_0^y + A_1^y x_t + A_2^y (p_t - \rho \bar{p}_t) + A_3^y \sigma_{c,t} \] (65)

The coefficients for are solved for in the same manner as the consumption coefficient, by expanding the pricing equation and collecting terms in each state variable. The amount of terms make the closed form solutions extremely complicated, so they are not reported here.

Expected return can then be calculated as in Bansal-Yaron 2004.

Appendix B - Cointegrated Consumption

Given the strong relation between levels of consumption and oil prices documented above, the changes in the behavior of oil prices should be reflected by changes in the underlying consumption processes. I therefore need to specify an endowment process that allows me to represent changes in persistence of oil prices in terms of the endowment process of oil. I also would like to allow the endowment process to directly reflect the predictive relation between oil prices and future growth. To do this I use a cointegrating framework similar to that of Hansen, Heaton, and Li (2008). Though it is important to emphasize that the results of the model do not rely on this cointegrating framework, and could be accomplished by allowing for a second \( x_t \) process relating to oil and affecting future growth.

The system of consumption dynamics implied by the equations (5) along with the
intratemporal cointegrating relation of equation (1) imply that the real oil price itself be a stationary variable, which is at odds with findings by Hamilton (2008) and Maslyuk and Smyth (2008). I reconcile this fact by noting that for the period from 1987 to 2003, augmented Dickey-Fuller and Phillips-Perron tests can reject the hypothesis of the unit root, though not for the whole sample (Table V). In addition, given the relation between oil prices and consumption, I can also approach this issue by looking for the existence of a cointegrating vector between oil consumption and consumption, or likewise by checking for the existence of two cointegrating vectors amongst the system of oil prices and both types of consumption. Johansen (1991) tests provide a method of testing a null hypothesis $H_0$: ($m$ cointegrating vectors) versus an alternative hypothesis $H_1$: ($m + 1$ cointegrating vectors). I perform tests for both the existence of a single cointegrating variable between oil consumption and aggregate consumption, as well as two cointegrating variables between aggregate consumption, oil consumption, and the real spot price. Results are reported in Panel A of Table XV.

The tests generally support the existence of a cointegrating vector between oil consumption and aggregate consumption, and the existence of two cointegrating variables in the trivariate system. Given the results of these estimates, I estimate a Vector Error Correction Model (VECM) of oil consumption and aggregate consumption in each subperiod of the following form, with results reported in Panel B of Table XV.

\[
\begin{bmatrix}
\Delta c_{t+1} \\
\Delta o_{t+1}
\end{bmatrix} = \begin{bmatrix}
\mu_c \\
\mu_o
\end{bmatrix} + \begin{bmatrix}
\pi_c \\
\pi_o
\end{bmatrix} e_t + \begin{bmatrix}
\Delta c_t \\
\Delta o_t
\end{bmatrix} + \Gamma_0 \begin{bmatrix}
\Delta c_{t-1} \\
\Delta o_{t-1}
\end{bmatrix} + \Gamma_1 \begin{bmatrix}
\Delta c_{t-2} \\
\Delta o_{t-2}
\end{bmatrix} + \Gamma_2 \begin{bmatrix}
\Delta c_{t-3} \\
\Delta o_{t-3}
\end{bmatrix}
\]  

(66)

This estimation supports the general form of consumption dynamics in the model. The negative loading of aggregate consumption growth on the cointegrating vector and the larger positive loading of oil consumption growth on this term are consistent with the model specification in a time of high mean reversion in oil prices. Additionally, the positive loadings on lagged aggregate consumption growth for both contemporaneous growth variables is consistent with a positive value of $\Phi_x$. For the second period the time series is too short for accurate inference from consumption data (only 19 quarters), so therefore I rely on the changes in the asset pricing data to choose the parameters of the growth process.
Table XV: VECM for Aggregate Consumption and Oil Consumption 1987Q1 - 2003Q3

Johansen tests of cointegration are conducted with four lags. Vector error correction model estimated using maximum likelihood. $c_t$ is the aggregation of nondurable and durable consumption, $o_t$ household energy consumption, $p_t$ is the log of the oil price. Consumption is real consumption per capita.

(a) Panel A: Johansen Tests

<table>
<thead>
<tr>
<th>Variables</th>
<th>Max Rank</th>
<th>Trace Stat</th>
<th>5% Critical Values</th>
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<td>17.89*</td>
<td>15.41</td>
</tr>
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<td>1</td>
<td>0.354</td>
<td>3.76</td>
</tr>
<tr>
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<td>29.68</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>17.072*</td>
<td>15.41</td>
</tr>
<tr>
<td></td>
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(b) Panel B: VECM

<table>
<thead>
<tr>
<th>Equation</th>
<th>Parameter</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>t-Value</th>
<th>Pr &gt;</th>
<th>Variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta c_{t+1}$</td>
<td>$\mu_c$</td>
<td>0.002*</td>
<td>0.001</td>
<td>2.32</td>
<td>0.02</td>
<td>$c_t - \alpha o_t$</td>
</tr>
<tr>
<td></td>
<td>$\pi_c$</td>
<td>-0.007*</td>
<td>0.003</td>
<td>-1.97</td>
<td>0.049</td>
<td>$c_t - \alpha o_t$</td>
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<tr>
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<td>3.1</td>
<td>0.002</td>
<td>$\Delta c_t$</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_1^{1,1}$</td>
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<td>0.137</td>
<td>0.51</td>
<td>0.611</td>
<td>$\Delta c_{t-1}$</td>
<td></td>
</tr>
<tr>
<td>$\Gamma_2^{1,1}$</td>
<td>0.321*</td>
<td>0.126</td>
<td>2.55</td>
<td>0.011</td>
<td>$\Delta c_{t-2}$</td>
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</tr>
<tr>
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<td>0.022</td>
<td>0.6</td>
<td>0.55</td>
<td>$\Delta o_t$</td>
<td></td>
</tr>
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<td>0.131</td>
<td>$\Delta o_{t-2}$</td>
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<tr>
<td>$\Delta o_{t+1}$</td>
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<td>0.955</td>
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<tr>
<td>$\Gamma_2^{2,1}$</td>
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<td>0.701</td>
<td>-0.22</td>
<td>0.827</td>
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<td>$\Gamma_0^{2,2}$</td>
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<td>$\Gamma_2^{2,2}$</td>
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<td>-1.32</td>
<td>0.185</td>
<td>$\Delta o_{t-2}$</td>
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</tbody>
</table>

Cointegrating Vector

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimate</th>
<th>Std. Error</th>
<th>t-Value</th>
<th>Pr &gt;</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$c_t$</td>
<td>1</td>
<td>1</td>
<td>0.002*</td>
<td>0.001</td>
<td>2.32</td>
</tr>
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<td>$o_t$</td>
<td>-0.007*</td>
<td>0.003</td>
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